Heavy quark bound states in the QGP phase*

⇒Original idea by Matsui and Satz: J/Ψ suppression as a signature of QGP;

⇒J/Ψ production cross section at SPS: NA50 data;

⇒Lattice findings for the meson spectral functions above T_c;

⇒An alternative approach: potential model calculation based on the lattice data for the color-singlet $Q\overline{Q}$ free energy;

⇒New experimental data: NA60 (SPS) and PHENIX (RHIC);

⇒Conclusions and perspectives.

*W.M. Alberico, A. B., A. De Pace and A. Molinari (Turin U. and INFN, Turin), submitted to Phys. Rev. D.
e-Print Archive: hep-ph/0507084.

Original idea by Matsui and Satz†

⇒Statement: the J/Ψ *anomalous suppression* in high energy AA collisions represents an unambiguous signature of deconfinement.

⇒Underlying assumptions:

•The J/Ψ are produced in the very early stage of the collision $\tau_{\text{form}} \approx 0.1$ fm/c;

•Crossing a deconfined medium the $c\bar{c}$ bound states tend to melt (Debye screening):

$$V(r) \sim -\frac{\alpha}{r} \to -\frac{\alpha}{r} e^{-m_D r}$$

•The heavy quarks hadronize by combining with light quarks only (secondary production is neglected)

⇒Questions:

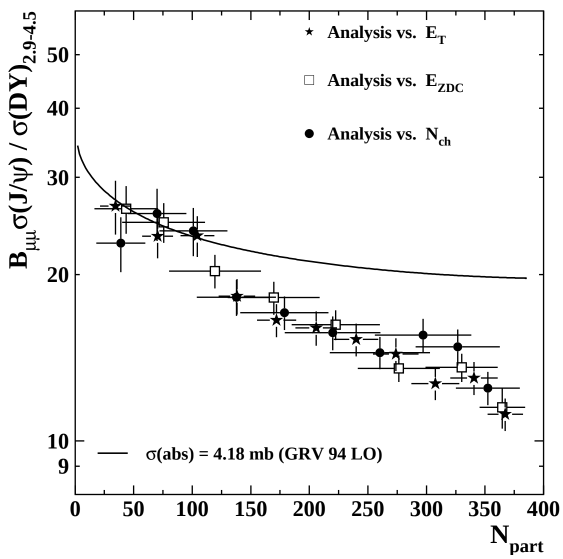
•Do all the $c\bar{c}$ states really melt above T_C or does the screened potential still support bound states?

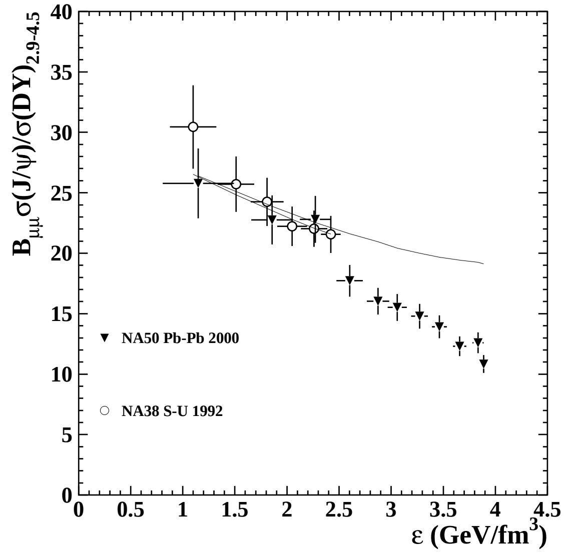
•Is recombination really negligible?

	SPS	RHIC	LHC
$\sqrt{s}$(GeV)	18	200	5500
$N_{c\bar{c}}$	0.2	10	200
N_{ch}	1350	3250	16500

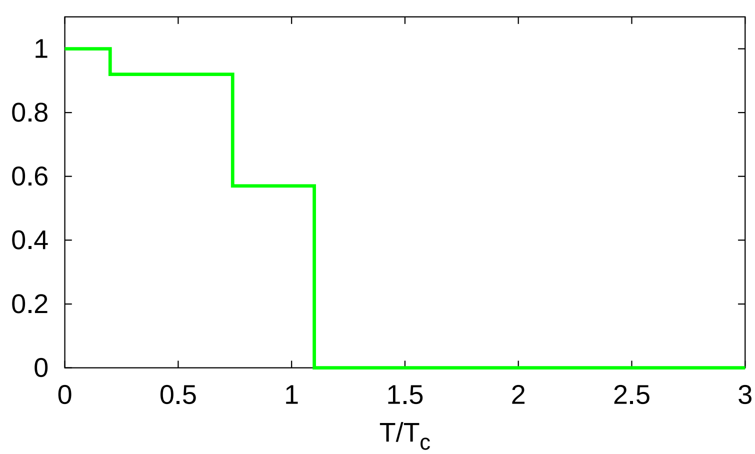
†T. Matsui and H. Satz, PLB 178 (1986).

NA50 Data and Sequential Charmonium Suppression‡





⇒continuous curve=normal nuclear matter absorption.



⇒Sequential suppression pattern: Ψ' (10%), χ_c (30%), J/Ψ (60%).

⇒Calculation done inserting the lattice $Q\overline{Q}$ free energy into the Schrödinger equation.

‡NA50 collaboration, EPJC 39 (2005)
S. Digal, P. Petreczky and H. Satz, PRD64.

Thermal meson correlation functions

⇒Current operator:

$$J_M(-i\tau,\boldsymbol{x}) = \bar{q}(-i\tau,\boldsymbol{x})\Gamma_M q(-i\tau,\boldsymbol{x}) \ ,$$

where $\Gamma_M = 1, \gamma^5, \gamma^\mu, \gamma^\mu\gamma^5$.

⇒*Thermal meson 2 point function*:

$$G_M(-i\tau,\boldsymbol{x}) = \langle \widetilde{J}_M(-i\tau,\boldsymbol{x})\widetilde{J}_M^\dagger(0,\boldsymbol{0})\rangle$$
$$= T\sum_{n=-\infty}^{+\infty}\int\frac{d^3p}{(2\pi)^3}e^{-i\omega_n\tau}e^{i\boldsymbol{p}\cdot\boldsymbol{x}}\chi_M(i\omega_n,\boldsymbol{p}) \ ,$$

⇒Spectral representation:

$$\chi_M(i\omega_n,\boldsymbol{p}) = -\int_{-\infty}^{+\infty}d\omega\frac{\sigma_M(\omega,\boldsymbol{p})}{i\omega_n-\omega} \ ,$$

where the thermal meson spectral function

$$\sigma_M(\omega,\boldsymbol{p}) = \frac{1}{\pi}\mathrm{Im}\,\chi_M(\omega+i\eta,\boldsymbol{p})$$

has been introduced.

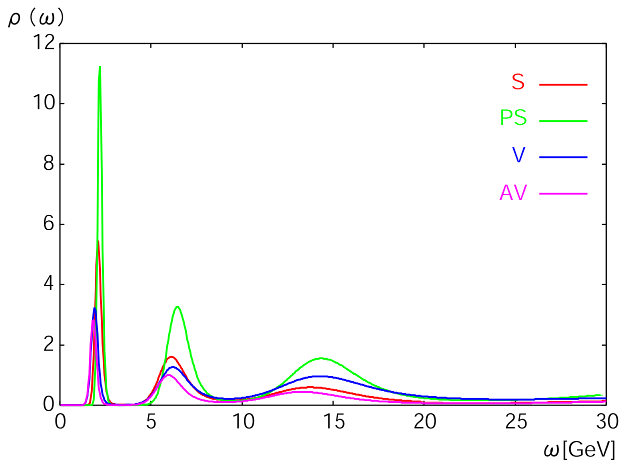
⇒Propagation along the (imaginary) temporal direction:

$$G_M(-i\tau,\boldsymbol{p}) = \int_0^{+\infty}d\omega \ \sigma_M(\omega,\boldsymbol{p})\frac{\cosh(\omega(\tau-\beta/2))}{\sinh(\omega\beta/2)} \ .$$

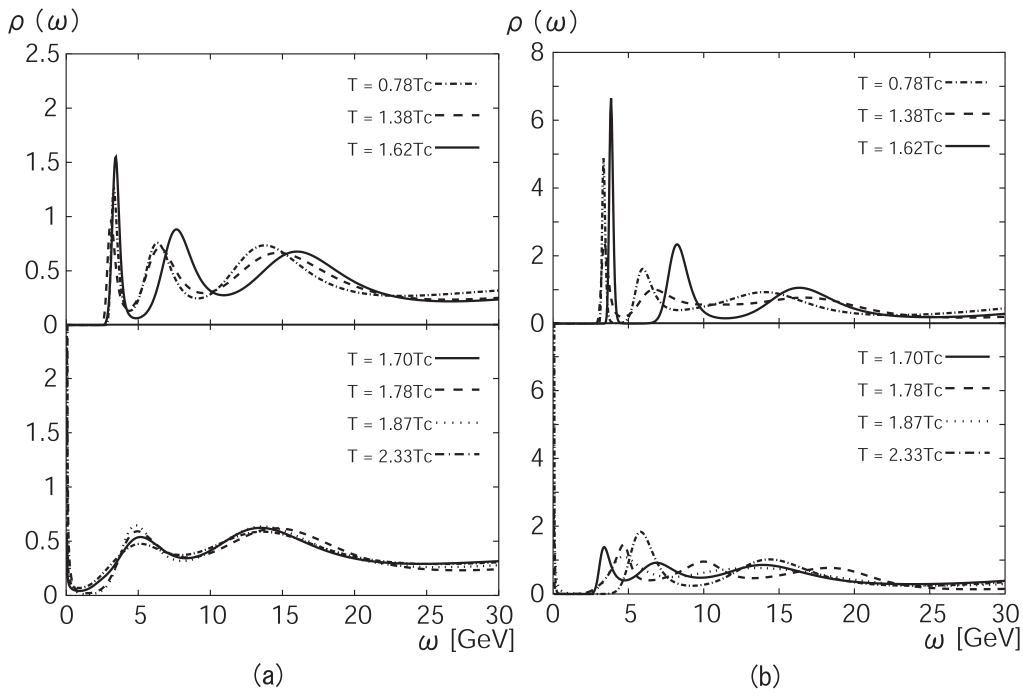
• $G_M(-i\tau,\boldsymbol{p}=0)$ is measured on the lattice for a finite set of values of τ (~ 20).

• $\sigma_M(\omega,\boldsymbol{0})$ has to be reconstructed (Maximum Entropy Method usually employed).

Lattice Meson Spectral Functions above T_C[§]



$\Rightarrow s\bar{s}$ dimensionless spectral function at $T = 1.38T/T_c$. Peak position at $\omega = 2.4m_\phi(T = 0)$.



$\Rightarrow J/\Psi$ (a) and η_c (b) dimensionless spectral functions. Shift of the peak position very small.

[§]T. Hatsuda, hep-lat/0509306 and references therein.

Polyakov Loop and Heavy Quark Free Energy

⇒Field operators

•$\psi_a(\boldsymbol{r},\tau)$ and $\psi_a^\dagger(\boldsymbol{r},\tau)$ destroying (creating) a heavy quark Q with color a at the point $(\vec{r},\tau)$.

•$\psi_a^c(\boldsymbol{r},\tau)$ and $\psi_a^{\dagger\,c}(\boldsymbol{r},\tau)$ playing the same role for the antiquark $\overline{Q}$;

⇒Evolution along the (imaginary) temporal direction:

$$\psi_i(\boldsymbol{r},\beta) = \mathrm{P}\exp\left(ig\int_0^\beta d\tau\, t^a\cdot A_0^a(\boldsymbol{r},\tau)\right)_{ij}\psi_j(\boldsymbol{r},0)$$

$$\psi_i^c(\boldsymbol{r},\beta) = \mathrm{P}\exp\left(ig\int_0^\beta d\tau\, \bar{t}^a\cdot A_0^a(\boldsymbol{r},\tau)\right)_{ij}\psi_j^c(\boldsymbol{r},0)\ ;$$

⇒Polyakov line:

$$W(\boldsymbol{r}) = \mathrm{P}\exp\left(ig\int_0^\beta d\tau\, t^a\cdot A_0^a(\boldsymbol{r},\tau)\right)\ ;$$

⇒Polyakov loop:

$$L(\boldsymbol{r}) \equiv \frac{1}{N_c}\mathrm{Tr}\,\mathrm{P}\exp\left(ig\int_0^\beta d\tau\, t^a\cdot A_0^a(\boldsymbol{r},\tau)\right) = \frac{1}{N_c}\mathrm{Tr}\,W(\boldsymbol{r})\ ;$$

⇒Free energy of a thermal bath with with a heavy static quark (antiquark) placed at $\boldsymbol{x}$ ($\boldsymbol{y}$):

$$e^{-\beta F_{Q\bar{Q}}(\boldsymbol{x};\boldsymbol{y})} = \frac{1}{N_c^2}\sum_s \langle s|e^{-\beta H}|s\rangle$$
$$= \frac{1}{N_c^2}\sum_{s'}\sum_{a,b}\langle s'|\psi_a(\boldsymbol{x},0)\psi_b^c(\boldsymbol{y},0)e^{-\beta H}\psi_a^\dagger(\boldsymbol{x},0)\psi_b^{\dagger c}(\boldsymbol{y},0)|s'\rangle \ ;$$

•$|s'\rangle$: states without heavy quark sources,
•$1/N_c^2$: average over the colors;

⇒$e^{-\beta H}$: Operator performing temporal translations, hence

$$e^{-\beta F_{Q\bar{Q}}(\boldsymbol{x};\boldsymbol{y})} = \frac{1}{N^2}\sum_{s'}\langle s'|e^{-\beta H}\psi_a(\boldsymbol{x},\beta)\psi_a^\dagger(\boldsymbol{x},0)\psi_b^c(\boldsymbol{y},\beta)\psi_b^{\dagger c}(\boldsymbol{y},0)|s'\rangle$$

⇒Temporal evolution given by the Polyakov line:

$$e^{-\beta F_{Q\bar{Q}}(\boldsymbol{x};\boldsymbol{y})} = \sum_{s'}\langle s'|e^{-\beta H}\frac{1}{N}\mathrm{Tr}W(\boldsymbol{x})\frac{1}{N}\mathrm{Tr}W^\dagger(\boldsymbol{y})|s'\rangle$$
$$= \mathrm{Tr}\left[e^{-\beta H}L(\boldsymbol{x})L^\dagger(\boldsymbol{y})\right] \ ;$$

⇒Free energy in absence of static sources:

$$e^{-\beta F_{00}} = \mathrm{Tr}\left[e^{-\beta H}\right] \ ;$$

⇒Change in the free energy:

$$e^{-\beta\Delta F_{Q\bar{Q}}(\boldsymbol{x};\boldsymbol{y})} = \frac{\mathrm{Tr}\left[e^{-\beta H}L(\boldsymbol{x})L^\dagger(\boldsymbol{y})\right]}{\mathrm{Tr}[e^{-\beta H}]} = \langle L(\boldsymbol{x})L^\dagger(\boldsymbol{y})\rangle \ ;$$

NB It is a *color averaged* quantity according to the decomposition:

$$\overline{N} \otimes N = 1 \oplus (N^2-1) \, ,$$

yielding (for the case $N=3$)

$$e^{-\beta F_{Q\bar{Q}}(r,T)} = \frac{1}{9}e^{-\beta F_1(r,T)} + \frac{8}{9}e^{-\beta F_8(r,T)} \, .$$

$Q\overline{Q}$ Free Energy on the Lattice

⇒Link Variable connecting the sites x and $x+a\hat{\mu}$:

$$U_\mu(x) \in SU(3) \qquad (\mu = 0\ldots 3)$$

⇒Polyakov line $W(\boldsymbol{x})$ → product of the link variables along the temporal direction:

$$W(\boldsymbol{x}) = \prod_{\tau=1}^{N_\tau} U_0(\boldsymbol{x},\tau)\ .$$

⇒Color singlet free energy of a (infinitely) heavy $Q\overline{Q}$ pair at a distance $r = |\boldsymbol{x}|$ in a thermal bath of gluons and N_f light dynamical quarks:

$$e^{-\beta F_1(r,T)+C(T)} = \frac{1}{3}\mathrm{Tr}\langle W(\boldsymbol{x})W^\dagger(\boldsymbol{0})\rangle\ .$$

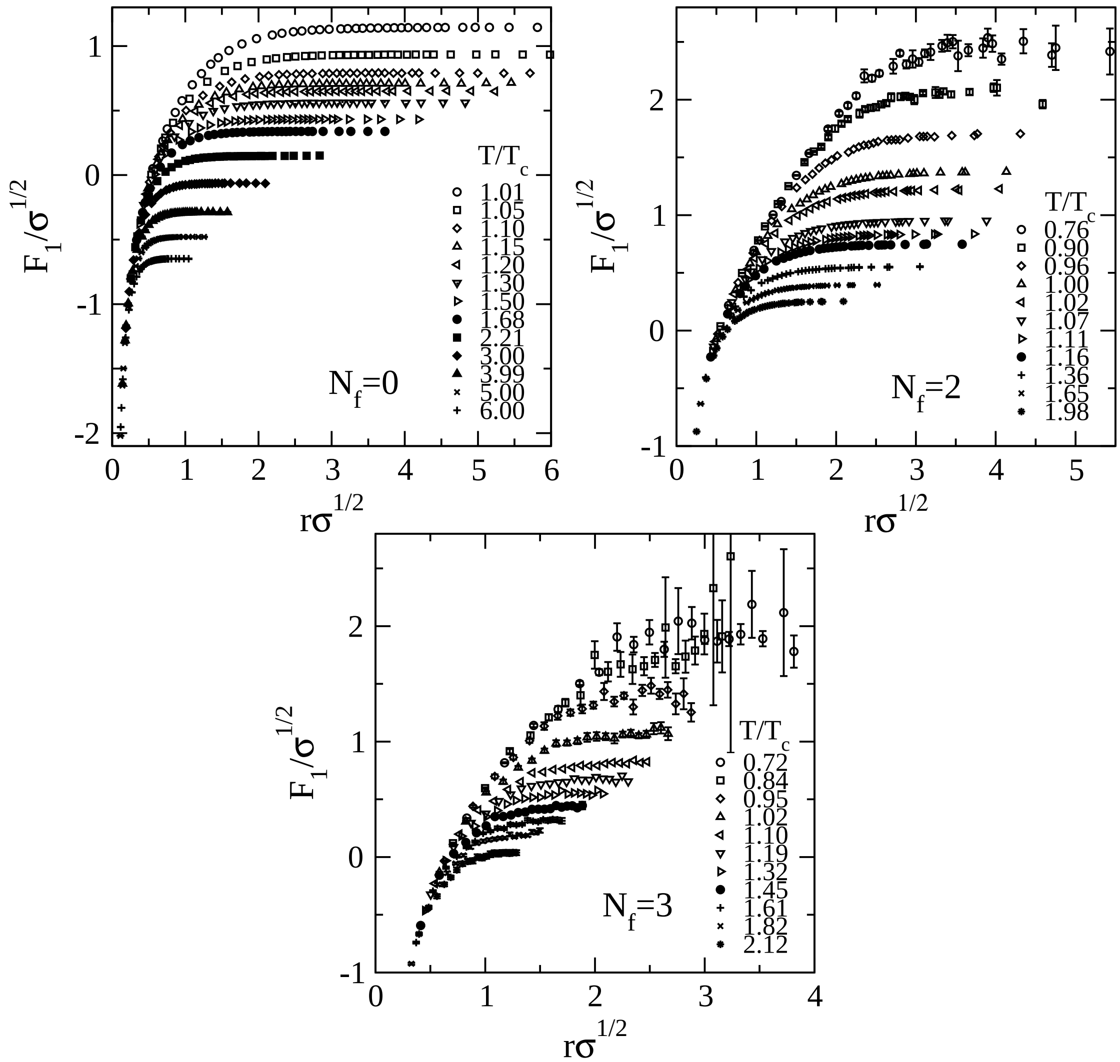
⇒For $r \ll 1/T$ the physics is not affected by the presence of the thermal bath → the constant $C(T)$ is fixed, for each value of T, by matching at the shortest distance available on the lattice $F_1(r,T)$ with the $T=0$ heavy quark potential:

$$F_1(r \ll 1/T, T) \equiv V(r, T=0)$$

⇒Usual Cornell-like parameterization for the $T=0$ potential:

$$\frac{V(r)}{\sqrt{\sigma}} = -\frac{4}{3}\frac{\alpha}{r\sqrt{\sigma}} + \sqrt{\sigma}r,$$

Lattice Data employed¶



⇒First two sets:

•$\alpha = 0.195(1)$ $(N_f = 0)$, $\alpha = 0.212(3)$ $(N_f = 2)$;

•$\sqrt{\sigma} = 420$ MeV

⇒Third set $(N_f = 3)$: $\sqrt{\sigma} \approx 460$ MeV.

¶O. Kaczmarek et al., PLB 543, 41 (2002);
O. Kaczmarek and F. Zantow, hep-lat/0503017;
P. Petreczky and K. Petrov, PRD70, 054503 (2004).

Parameterization of the Lattice Data

⇒Limiting behaviour:

•Very short distance ($r \ll 1/T$): *perturbative one-gluon exchange*.

$$F_1(r,T) \underset{rT \ll 1}{\sim} -\frac{4}{3}\frac{\alpha(r)}{r}$$

Thermal effects are negligible, the running of the coupling is set by the $Q\overline{Q}$ distance.

•High temperature and ($T \gg T_c$) large distance ($rT \gg 1$): exchange of a *resummed electrostatic gluon*.

$$F_1(r,T) \underset{rT \gg 1}{\sim} -\frac{4}{3}\frac{\alpha(T)}{r} e^{-m_D(T) r} + F_1(r=\infty,T)$$

$\alpha = \alpha(T)$ runs with the temperature, $m_D(T)$ is the Debye screening mass (dressing of the electrostatic gluon).

⇒In both limits the running of the coupling is given by a *Renormalization Group Equation* (RGE) $\rightarrow \alpha = \alpha(\mu)$ with

•$\mu \sim 1/r$ (short distance);

•$\mu \sim T$ (high temperature and large distance).

⇒Functional form of the free energy (Ansatz I):

$$F_1(r,T) = -\frac{4}{3}\frac{\alpha(r,T)}{r}e^{-M(T)r} + C(T).$$

⇒Running of the coupling (Ansatz II):

$$\alpha(r,T) = \alpha(\mu = c_r/r + c_t T)$$

⇒Our parameterization:

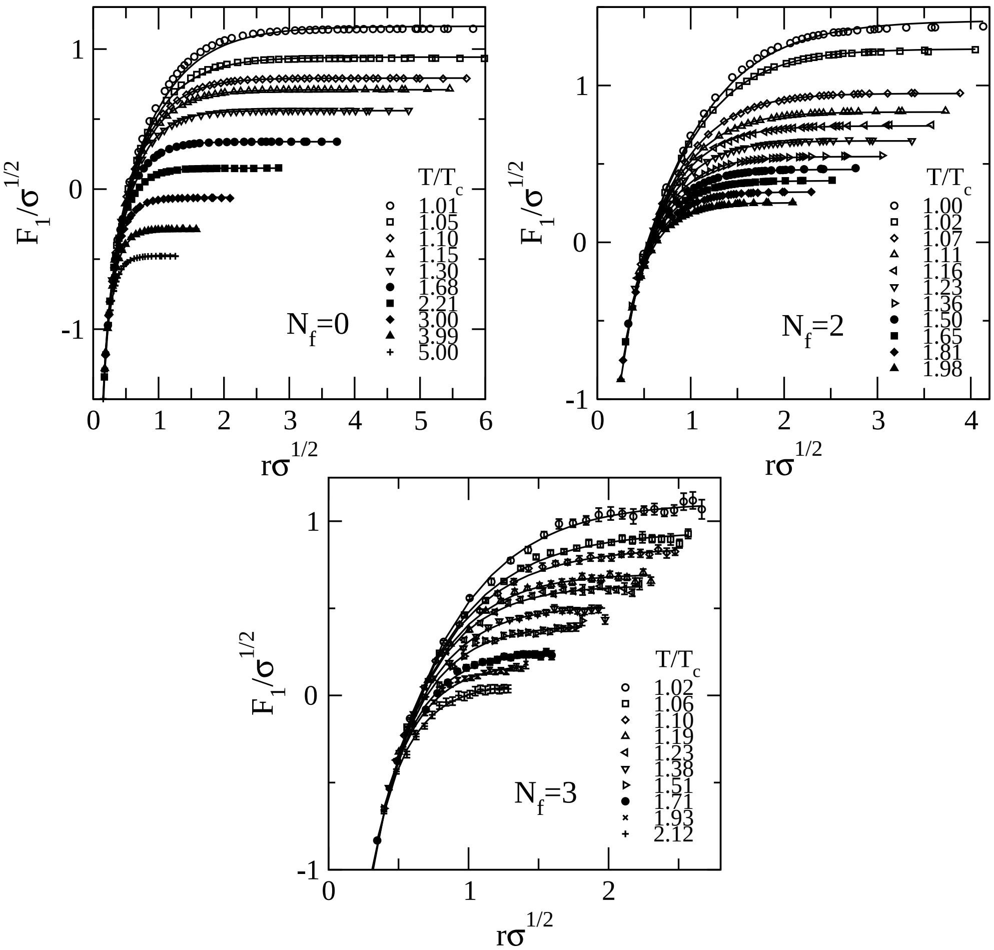
$$y = -\frac{4}{3}\frac{\alpha(\tilde{\mu})}{x}e^{-a_3 x} + a_0 \quad \text{with} \quad \tilde{\mu} = \frac{a_1}{x} + a_2\ ,$$

where $y = F_1/\sqrt{\sigma}$ and $x = r\sqrt{\sigma}$ and $\alpha(\tilde{\mu})$ solution of the RGE with the 2-loop beta function.

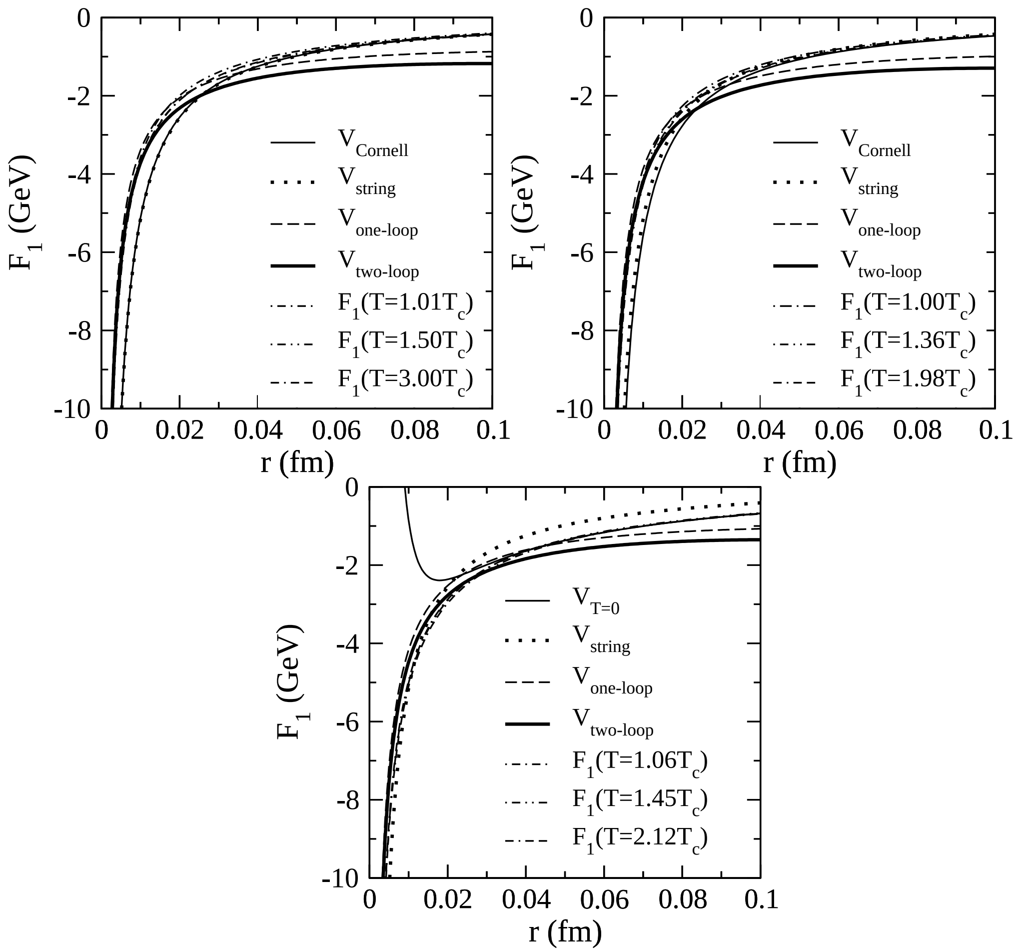
⇒Fitting procedure:

- for each T the four coefficients a_i have been determined;
- a weighted average of the values of a_1 has been performed getting $a_1 = 0.2719(2)$ $(N_f = 0)$, $a_1 = 0.2687(7)$ $(N_f = 2)$ and $a_1 = 0.2354(17)$ $(N_f = 3)$.
- the data have been fitted again keeping a_1 fixed.

⇒$\chi^2/dof \sim 1$ for all the temperatures considered.



F_1/σ^{1/2}
rσ^{1/2}
T/T_c
1.01
1.05
1.10
1.15
1.30
1.68
2.21
3.00
3.99
5.00
N_f=0
1.00
1.02
1.07
1.11
1.16
1.23
1.36
1.50
1.65
1.81
1.98
N_f=2
1.02
1.06
1.10
1.19
1.23
1.38
1.51
1.71
1.93
2.12
N_f=3



F_1 (GeV)
r (fm)
V_Cornell
V_string
V_one-loop
V_two-loop
F_1(T=1.01T_c)
F_1(T=1.50T_c)
F_1(T=3.00T_c)
F_1(T=1.00T_c)
F_1(T=1.36T_c)
F_1(T=1.98T_c)
V_T=0
F_1(T=1.06T_c)
F_1(T=1.45T_c)
F_1(T=2.12T_c)

$Q\overline{Q}$ Potential energy

⇒Lattice data provide the $Q\overline{Q}$ free energy:

$$F = U - TS\;;$$

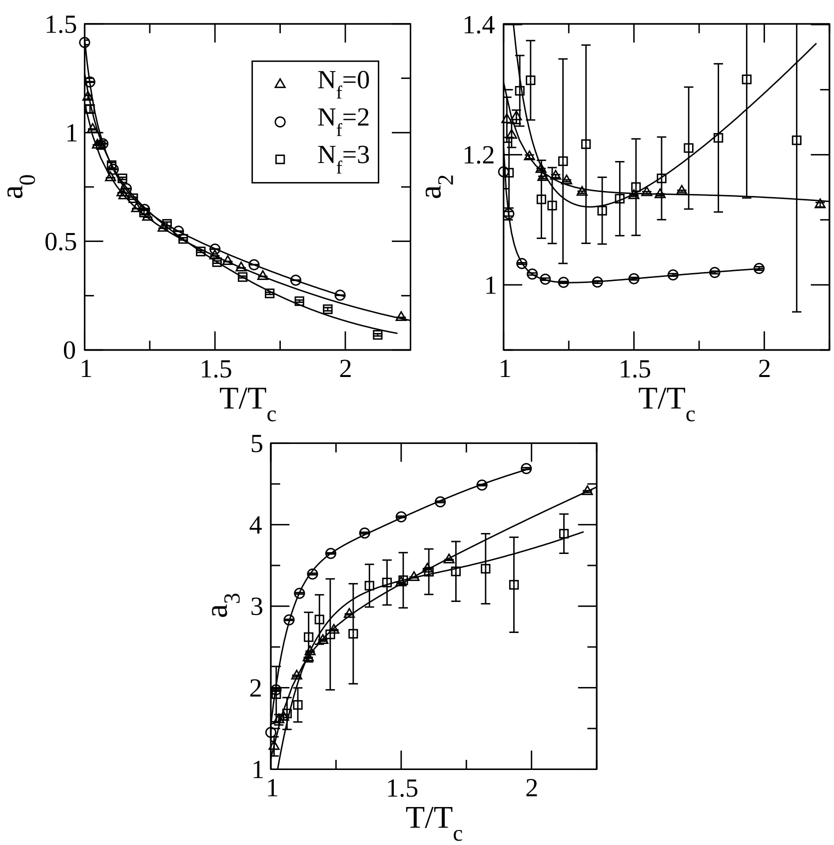
⇒It is possible to extract the internal energy (=potential energy for infinitely heavy sources)

$$U = -T^2 \frac{\partial (F/T)}{\partial T}$$

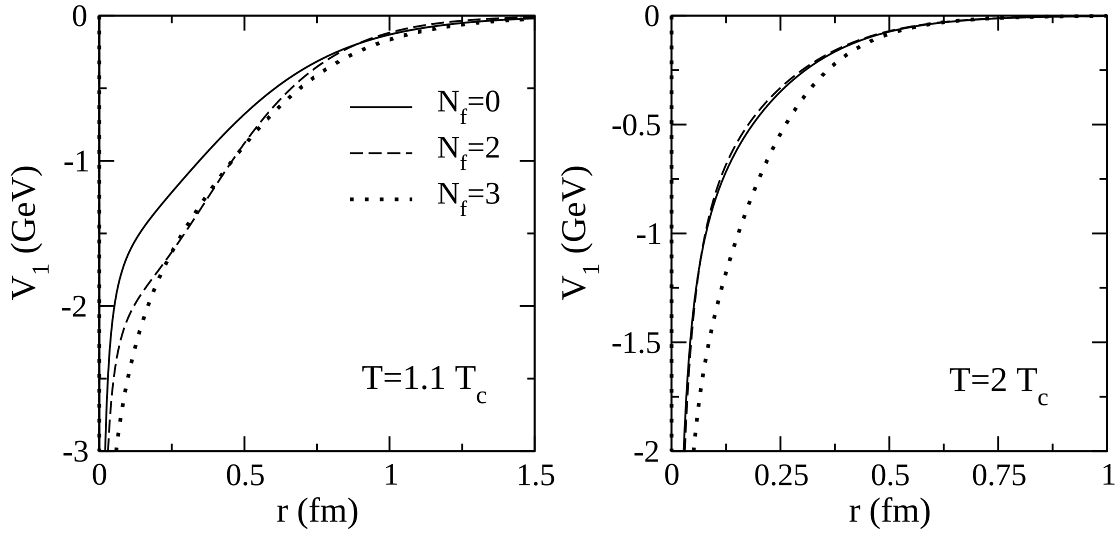
and the entropy

$$S = -\frac{\partial F}{\partial T}\;;$$

⇒The T dependence of the fitting coefficients is needed in order to get $U(r,T)$:



$Q\overline{Q}$ Bound states in the QGP



⇒Effective potential

$$V_1(r,T) = U_1(r,T) - U_1(r \to \infty, T) \;;$$

⇒Schrödinger equation

$$\left[-\frac{\boldsymbol{\nabla}^2}{2\mu} + V_1(r,T)\right]\psi(\boldsymbol{r},T) = \varepsilon(T)\psi(\boldsymbol{r},T) \;,$$

μ being the *reduced mass*;

⇒Quarkonium mass

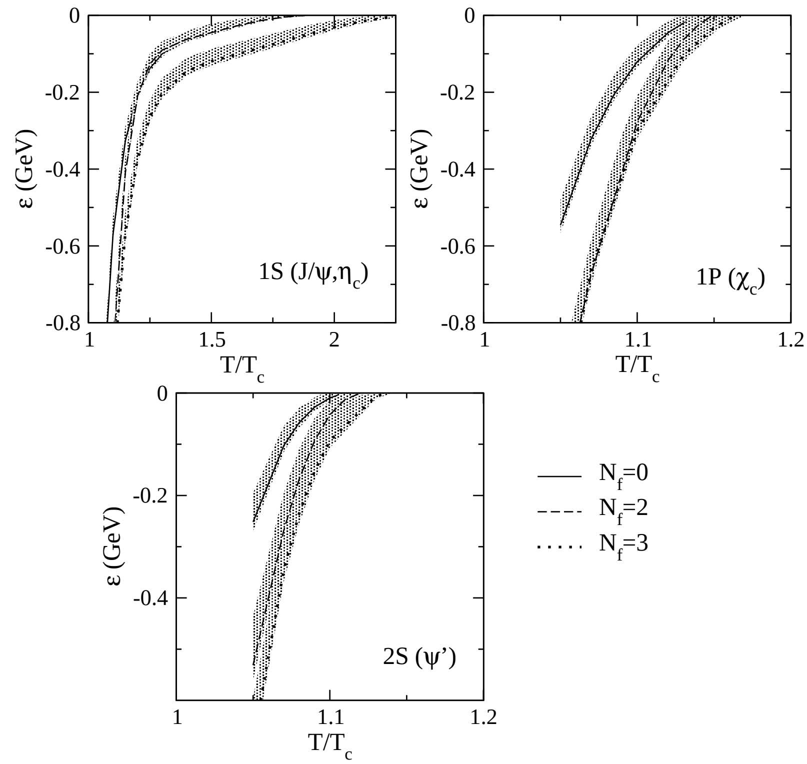
$$M(T) = 2m_{c(b)} + \epsilon(T) + U_1(r \to \infty, T) \;;$$

⇒Quark masses from the PDG:

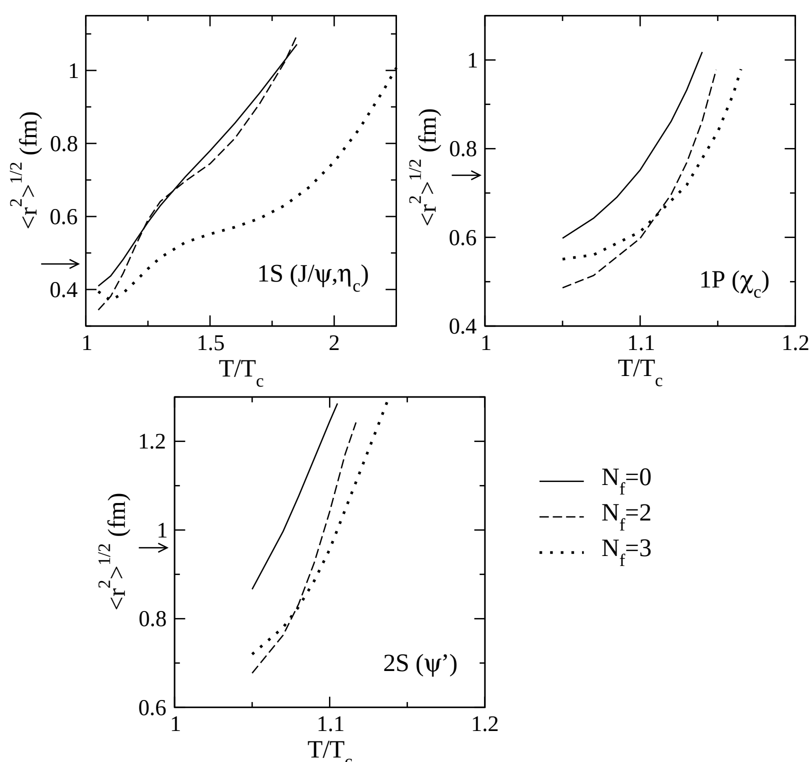
- $1.15 < m_c < 1.35$ GeV,
- $4.1 < m_b < 4.4$ GeV.

⇒Charmonium states in the QGP

- Binding energies:

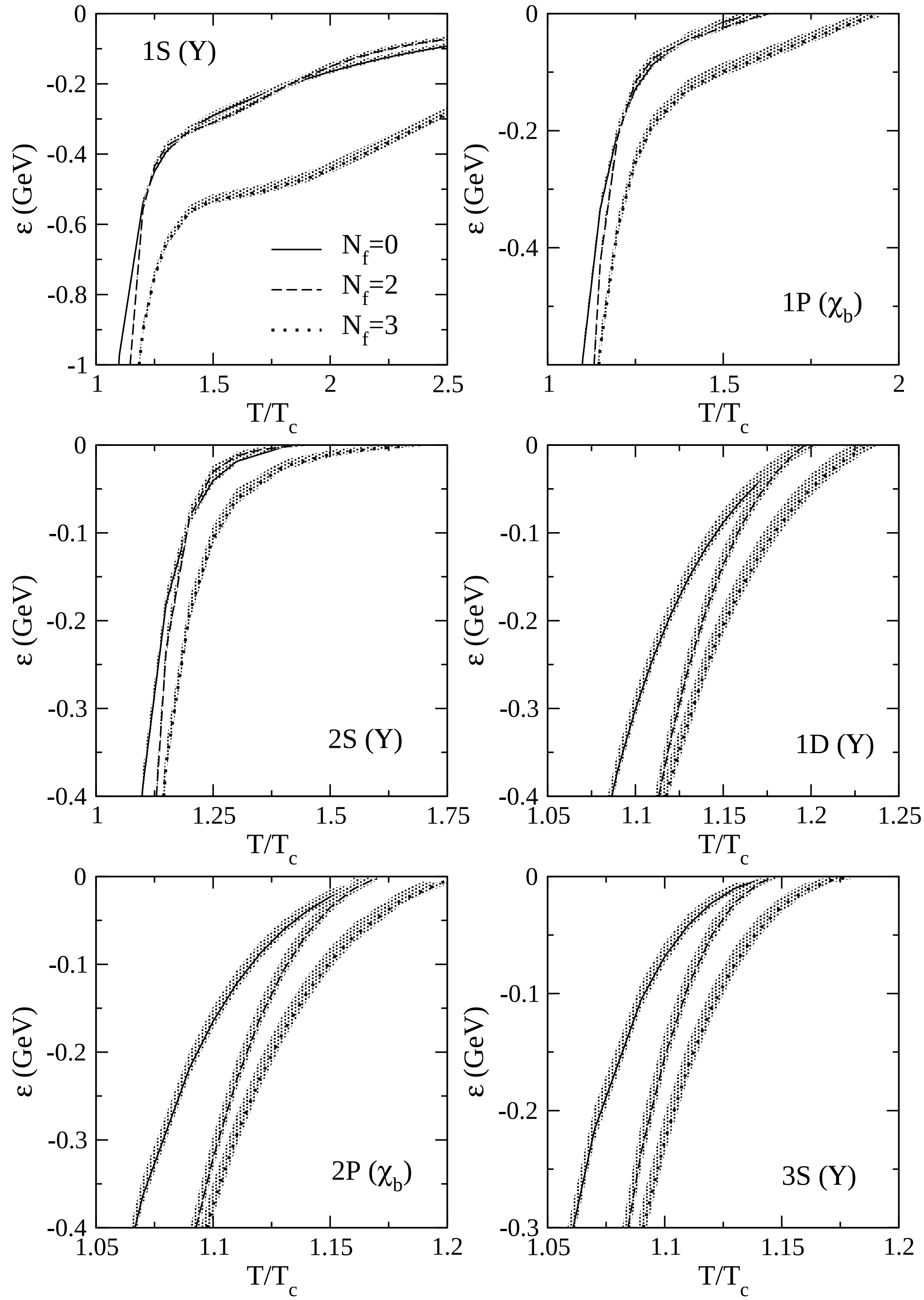


- Mean square radii:

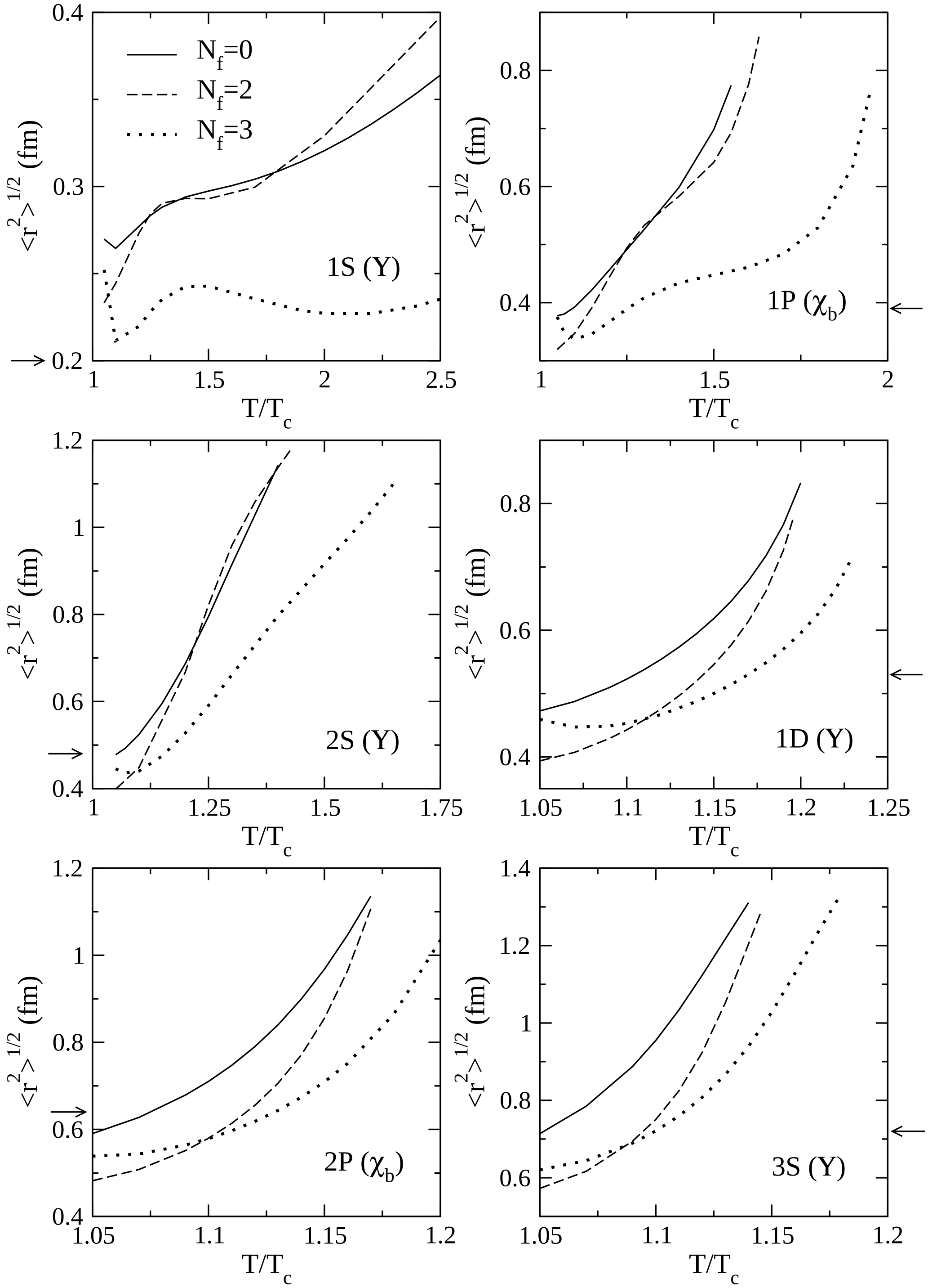


⇒Bottomonium states in the QGP

- Binding energies:

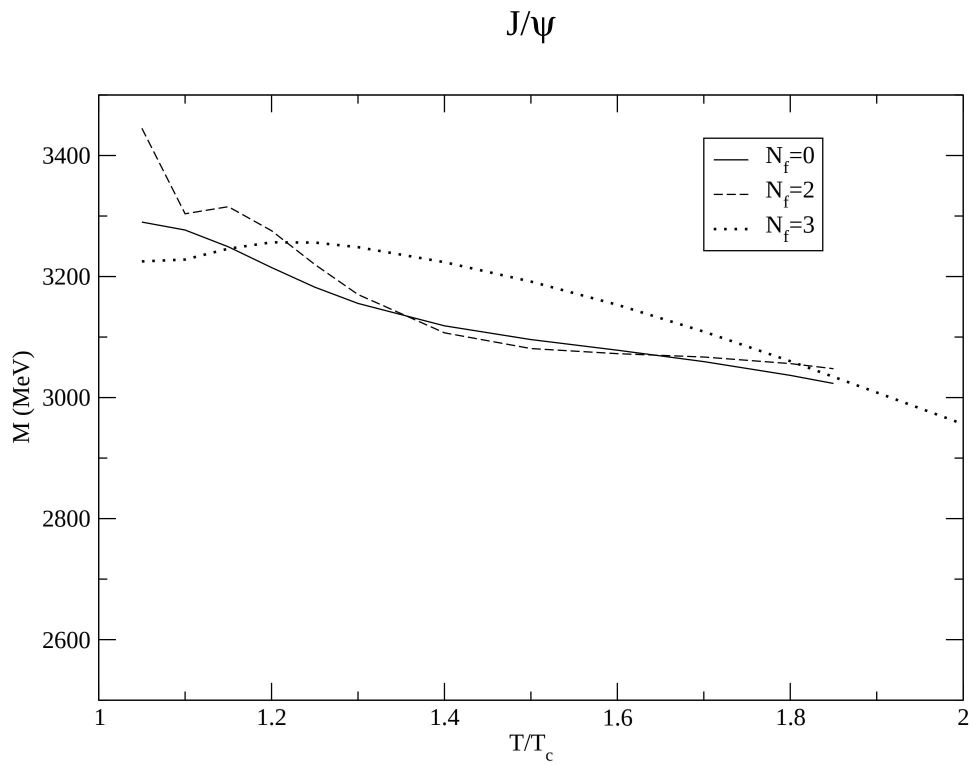


● Mean square radii:

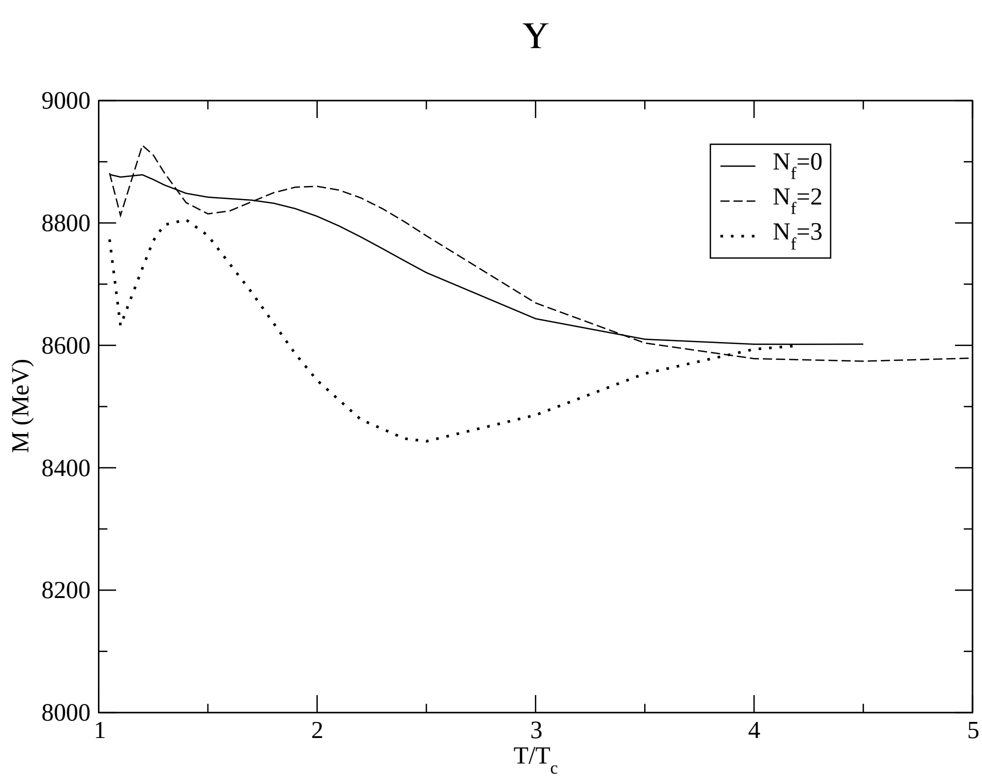


⇒Quarkonium masses:

•Charmonium 1S ($m^{T=0} = 3097$ MeV)



•Bottomonium 1S ($m^{T=0} = 9460$ MeV)



⇒*Extracting the potential energy $U_1(r,T)$ from the free energy $F_1(r,T)$ is crucial*!

In previous studies $F_1(r,T)$ was inserted directly into the Schrödinger equation as an effective potential‖. This turned out to underestimate the *dissociation temperature* T_d.

	J/Ψ	$\chi_c(1P)$	$\Psi'(2S)$
T_d/T_c	1.10	0.74	0.1-0.2

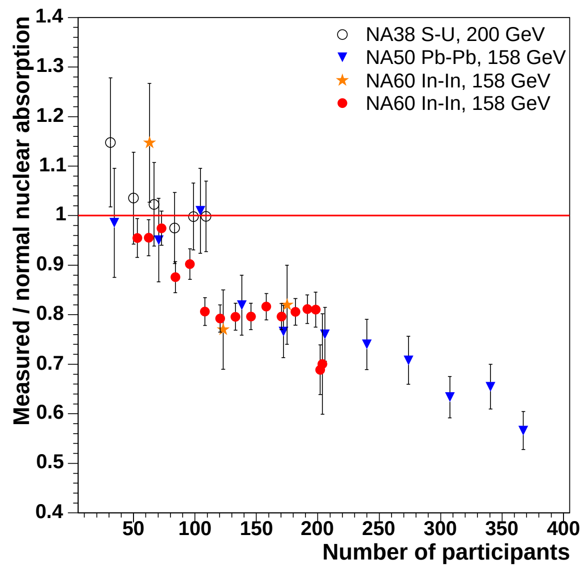
Employing $U_1(r,T)$ leads to a dissociation pattern in agreement with the lattice spectral functions results.

	$T_d/T_c\,(N_f=0)$	$T_d/T_c\,(N_f=2)$	$T_d/T_c\,(N_f=3)$
J/Ψ	1.75-1.95	1.78-1.92	2.15-2.35
χ_c	1.13-1.15	1.14-1.15	1.15-1.17
Ψ'	1.10-1.11	1.11-1.12	1.12-1.14
$\Upsilon(1S)$	4.4-4.7	6.4-6.8	4.1-4.3
χ_b	1.55-1.61	1.60-1.65	1.90-1.98
$\Upsilon(2S)$	1.40-1.45	1.38-1.46	1.62-1.72
$\Upsilon(1D)$	1.19-1.20	1.19-1.20	1.23-1.24
χ_b'	1.17-1.18	1.18-1.19	1.20-1.21
$\Upsilon(3S)$	1.14-1.15	1.14-1.15	1.17-1.18

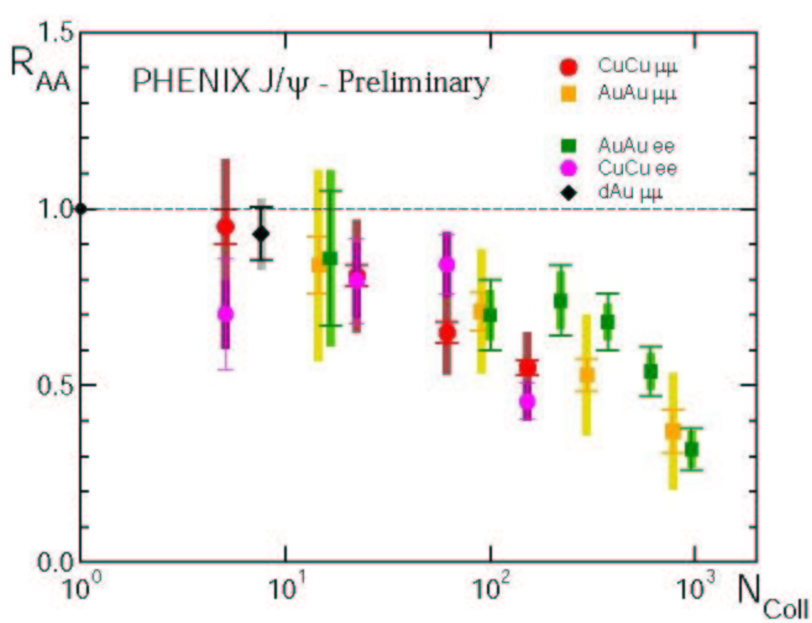
‖S. Digal, P. Petreczky and H. Satz, Phys. Rev. D 64, 094015 (2001).

Latest Experimental Results

⇒NA60** and previous data (NA38 and NA50)



⇒PHENIX††: $R_{AA} = \frac{dN/dy(AA)}{N_{\text{Coll}} dN/dy(pp)}$



**R. Arnaldi talk at QM2005.

††Y. Akiba, nucl-ex/0510008

Conclusions and perspectives

⇒Disentangling the potential energy from $F_1(r,T)$ seems to be necessary in order to get results in agreement with the lattice spectral function findings;

⇒A recent analysis of the SPS and RHIC[‡‡] data seems to indicate that *$\sim$ 60% of J/Ψ survive even at the highest enegy density reached at RHIC*, in agreement with our calculation: only the feed-down contribution is suppressed.

⇒Th: gluon dissociation and recombination cross sections can be evaluated and inserted into a kinetic rate equation.

⇒Exp: measuring the flow of the J/Ψ at RHIC would allow to understand whether the measured J/Ψ come from primordial production or from recombination.

[‡‡] See Satz's talk at the last CERN Heavy Ion Forum.