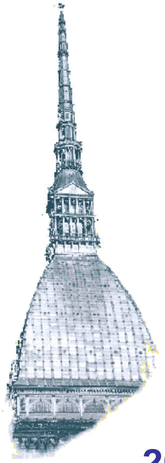
# Intrinsic motion effects in Deep Inelastic Scattering

Christian Türk

Dipartimento di Fisica, Universit`a di Torino.

INFN, Sezione di Torino



Strangeness, **Spin** and QGP 2005

Villa Gualino, Turin, Italy

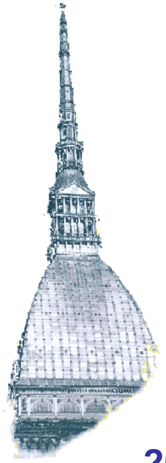
**European Graduate School**

*“Complex System of Hadrons and Nuclei”*

(Copenhagen, Giessen, Helsinki, Jyväskylä, Torino)

# Keywords

- ✓ Deep Inelastic Scattering (DIS)
- ✓ Semi Inclusive DIS (SIDIS)
- ✓ Intrinsic motion $\mathbf{k}_\perp$ $\mathbf{p}_\perp$
- ✓ Cahn effect (azimuthal dependence)
- ✓ Model parameters



# Motivations

- Lectures of M. Anselmino and R. Bertini.
- To understand the substructure of the nucleon in terms of the spin properties.
- Test PQCD in the spin sector.
- Active research interplay between theory and experiments.

# Basic DIS kinematics

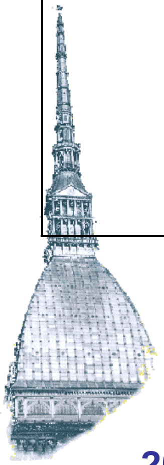
$$lp \rightarrow lX$$

$$s = (P + l)^2 , \quad Q^2 = -q^2$$

$$(P + q)^2 = W^2 = \frac{1 - x_b}{x_b} Q^2 + m_p^2$$

$$x_b = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{W^2 + Q^2 - m_p^2}$$

$$y = \frac{P \cdot q}{P \cdot l} = \frac{Q^2}{x_b \left(s - m_p^2\right)}$$



**Trento conventions** - *Phys. Rev.* ***D70*** *(2004) 117504*

# Definitions with intrinsic $\mathbf{k}_\perp$

$$x = \frac{k^-}{P^-}, \quad \mathbf{k}_\perp = k_\perp (\cos\varphi, \sin\varphi, 0)$$

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$$\hat{s} = xs - 2\mathbf{l}\cdot\mathbf{k}_\perp - k_\perp^2 \frac{x_b}{x}\left(1 - \frac{x_b s}{Q^2}\right)$$

$$\hat{t} = -Q^2$$

$$\hat{u} = -x\left(s - \frac{Q^2}{x_b}\right) + 2\mathbf{l}\cdot\mathbf{k}_\perp - k_\perp^2 \frac{x_b^2}{xQ^2}$$

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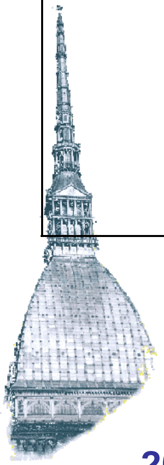
$$x = \frac{1}{2}x_b\left(1 + \sqrt{1 + 4\frac{k_\perp^2}{Q^2}}\right)$$

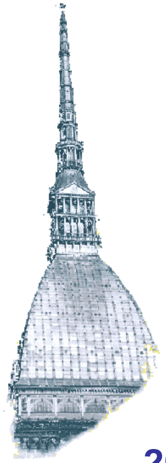


# Cross section with intrinsic $\mathbf{k}_\perp$

For the inclusive process in the QCD parton model

$$\frac{\mathrm{d}^2\sigma^{lp\to lX}}{\mathrm{d}x_b\mathrm{d}Q^2} = \frac{\pi\alpha^2}{Q^2}\frac{1}{x_b^2 s^2} L_{\mu\nu} W^{\mu\nu}$$

$$L_{\mu\nu} = 2\left(l_\mu l'_\nu + l'_\mu l_\nu - \eta_{\mu\nu} l \cdot l'\right)$$



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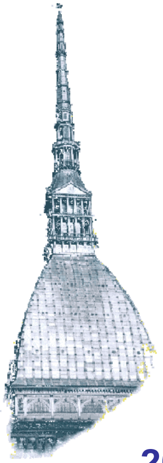


$$W^{\mu\nu} = \sum_q \int \mathrm{d}x\mathrm{d}^2\mathbf{k}_\perp \left(\frac{1}{x}\right) f_q\left(x, k_\perp\right) w^{\mu\nu}$$

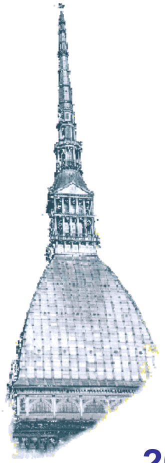
$$L_{\mu\nu}w^{\mu\nu} = 2e_q^2\delta\left(2q\cdot k - Q^2\right)\left(\hat{s}^2 + \hat{u}^2\right)$$

# Cross section with intrinsic $\mathbf{k}_\perp$

Cross section for general non collinear case

$$\frac{\mathrm{d}^2\sigma^{lp\to lX}}{\mathrm{d}x_b\mathrm{d}Q^2} = \sum_q \int \mathrm{d}^2\mathbf{k}_\perp f_q(x,k_\perp)\frac{\mathrm{d}\hat{\sigma}^{lq\to lq}}{\mathrm{d}Q^2}J(x_b,Q^2,\mathbf{k}_\perp)$$

$$\frac{\mathrm{d}\hat{\sigma}^{lq\to lq}}{\mathrm{d}Q^2} = e_q^2\frac{2\pi\alpha^2}{\hat{s}^2}\frac{\hat{s}^2+\hat{u}^2}{Q^4} \qquad J = \frac{\hat{s}^2}{x_b^2 s^2}\frac{x_b}{x}\left(1+\frac{x_b^2}{x^2}\frac{k_\perp^2}{Q^2}\right)^{-1}$$



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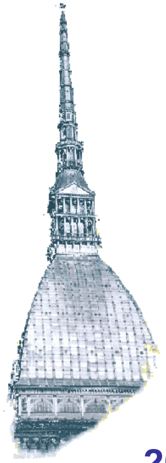
The collinear case ($\mathbf{k}_\perp = 0$) gives J = 1

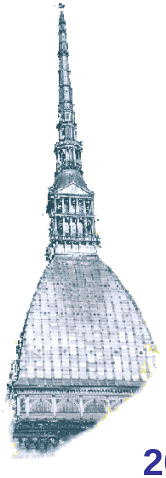


$$\frac{\mathrm{d}^2\sigma^{lp\to lX}}{\mathrm{d}x_b\mathrm{d}Q^2} = \sum_q f_q(x_b)\frac{\mathrm{d}\hat{\sigma}^{lq\to lq}}{\mathrm{d}Q^2} \qquad f_q\left(x,k_\perp\right) = f_q(x)\delta^2(\mathbf{k}_\perp)$$

# Why this cross section?

- If the final quark could be detected, reconstructing the current fragmentation jet, the cross section would be

$$\frac{d^5\sigma^{lp\to lX}}{dx_b\,dQ^2\,d^2\mathbf{k}_\perp} = \sum_q f_q(x,k_\perp)\frac{d\hat{\sigma}^{lq\to lq}}{dQ^2}J(x_b,Q^2,\mathbf{k}_\perp)$$



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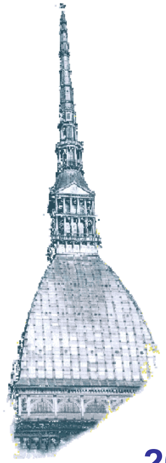
- Could test the azimuthal dependence $\varphi$ embedded in the expressions of the elementary Mandelstam variables

$$\mathbf{l}\cdot\mathbf{k}_\perp = Ek_\perp\sin\theta\cos\varphi$$

and was suggested by Cahn in SIDIS with the fragmentation process being essentially collinear in the direction of detection.



R.N. Cahn - *Phys. Lett.* ***B78*** *(1978) 269, Phys. Rev.* ***D40*** *(1989) 3107*

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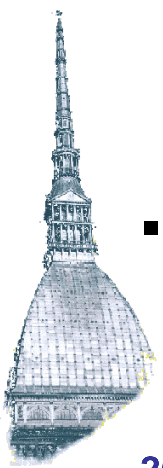
- We consider SIDIS process with taking into account **the intrinsic motion** and **the angular dependence** in the fragmentation process.

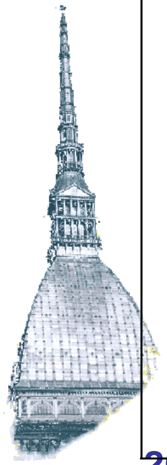


R.N. Cahn - *Phys. Lett.* ***B78*** *(1978) 269, Phys. Rev.* ***D40*** *(1989) 3107*

# Modifications for SIDIS

$$\frac{\mathrm{d}^7\sigma^{lp\to lX}}{\mathrm{d}x_b\mathrm{d}Q^2\mathrm{d}^2\mathbf{k}_\perp\mathrm{d}z\mathrm{d}^2\mathbf{p}_\perp} = \sum_q f_q(x,k_\perp)\frac{\mathrm{d}\hat{\sigma}^{lq\to lq}}{\mathrm{d}Q^2}J(x_b,Q^2,\mathbf{k}_\perp)D_q^h(z,p_\perp)$$

$$lp \to lhX$$



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$$z = \frac{\tilde{P}_h^+}{\tilde{k}'^+} \quad \mathbf{P}_T = P_T(\cos\phi_h, \sin\phi_h, 0)$$

$$\mathbf{p}_\perp = \mathbf{P}_h - \left(\mathbf{P}_h\cdot\hat{\mathbf{k}}'\right)\hat{\mathbf{k}}'$$

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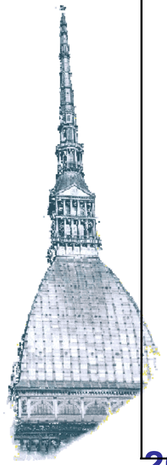
$$\mathbf{p}_\perp = \mathbf{P}_T - z_h\mathbf{k}_\perp + \mathcal{O}\left(\frac{k_\perp^2}{Q^2}\right)$$

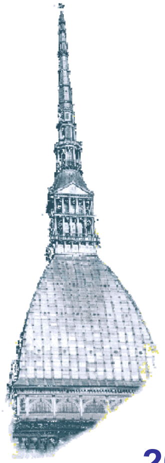
$$z = z_h + \mathcal{O}\left(\frac{k_\perp^2}{Q^2}\right) \quad z_h = \frac{P\cdot P_h}{P\cdot q}$$



# An instructive simple limit

Limit which only terms of order ($k_\perp$/Q) are retained.

$$x = x_b \quad z = z_h \quad \mathbf{p}_\perp = \mathbf{P}_T - z_h \mathbf{k}_\perp$$



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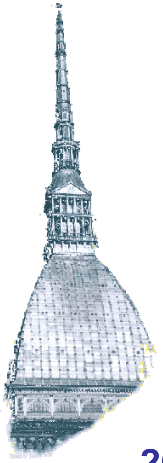
$$x = x_b \quad z = z_h \quad \mathbf{p}_\perp = \mathbf{P}_T - z_h \mathbf{k}_\perp$$

$$\hat{s}^2 = \frac{Q^4}{y^2}\left[1 - 4\frac{k_\perp}{Q}\sqrt{1-y}\cos\varphi\right]$$

$$\hat{u}^2 = \frac{Q^2}{y^2}(1-y)^2\left[1 - 4\frac{k_\perp}{Q}\frac{\cos\varphi}{\sqrt{1-y}}\right]$$

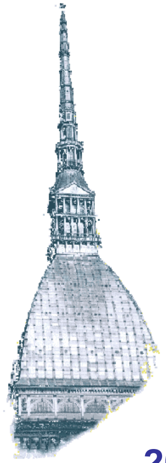
$$f_q(x,k_\perp) = f_q(x)\frac{1}{\pi\langle k_\perp^2\rangle}e^{-k_\perp^2/\langle k_\perp^2\rangle} \qquad D_q^h(z,p_\perp) = D_q^h(z)\frac{1}{\pi\langle p_\perp^2\rangle}e^{-p_\perp^2/\langle p_\perp^2\rangle}$$



# The approximate result

$$\frac{\mathrm{d}^5\sigma^{lp\to lhX}}{\mathrm{d}x_b\mathrm{d}Q^2\mathrm{d}z_h\mathrm{d}^2\mathbf{P}_T} = \sum_q \frac{2\pi\alpha^2 e_q^2}{Q^4} f_q(x_b) D_q^h(z_h) \Bigg[1 + (1-y)^2 - 4\frac{(2-y)\sqrt{1-y}\langle k_\perp^2\rangle z_h P_T}{\langle P_T^2\rangle Q}\cos\phi_h\Bigg] \frac{1}{\pi\langle P_T^2\rangle} e^{-P_T^2/\langle P_T^2\rangle}$$

$$\langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h^2\langle k_\perp^2\rangle$$



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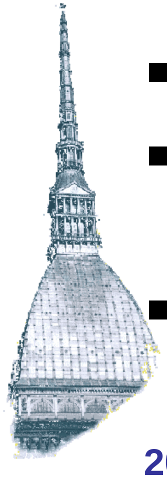
Observable

$$\langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h^2\langle k_\perp^2\rangle$$



- Unpolarized cross section shows a $\phi_h$ dependence.
- Dependence related to the intrinsic motion $k_\perp$, vanish for $k_\perp = 0$.
- Result holds at $P_T \sim \Lambda_{QCD} \sim k_\perp$.

# Cahn effect

- A way to study the azimuthal dependence in unpolarized SIDIS
- Experimental information on the average intrinsic motion.
  - ✓ Use several experimental data sets which exploits the azimuthal angle $\phi_h$ and the transverse momentum $P_T$.
  - ✓ Fit values on the introduced parameters.

$\langle k_\perp^2 \rangle$ $\langle p_\perp^2 \rangle$

# Data with $\phi_h$ dependence

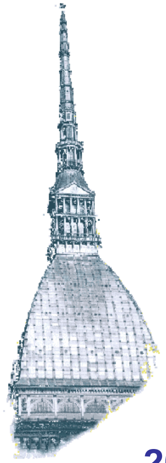
- EMC - 280 GeV muons scattering against a hydrogen target.

Observable:

$$\frac{\mathrm{d}\sigma^{lp\to lhX}}{\mathrm{d}\phi_h} = \int \mathrm{d}x_b \mathrm{d}Q^2 \mathrm{d}z_h \mathrm{d}P_T P_T \frac{\mathrm{d}^5\sigma^{lp\to lhX}}{\mathrm{d}x_b \mathrm{d}Q^2 \mathrm{d}z_h \mathrm{d}^2\mathbf{P}_T}$$

Cuts:

$$x_F > 0.1 \quad P_T > 0.2 \text{ GeV} \quad y < 0.8 \quad Q^2 > 4 \text{ GeV}^2$$



$$x_F = 2\frac{P_L}{W}$$

# Data with $P_T$ dependence

- EMC - $\mu p$ and $\mu d$ scattering at beam energy between 100 and 280 GeV.

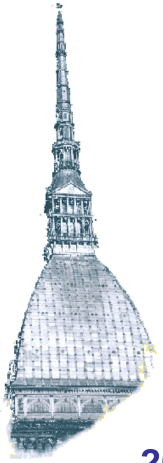
Observable:

$$\frac{1}{\sigma_{DIS} \frac{d\sigma}{dP_T^2}} = \frac{1}{2\sigma_{DIS}} \int d\phi_h dx_b dQ^2 dz_h \frac{d^5\sigma^{lp \to lhX}}{dx_b dQ^2 dz_h d^2\mathbf{P}_T}$$

Cuts:

$$Q^2 > 5\ \mathrm{GeV}^2 \quad W^2 < 90\ \mathrm{GeV}^2 \quad E_h > 5\ \mathrm{GeV}$$

$$0.1 < z_h < 1.0 \quad 0.2 < y < 0.8$$

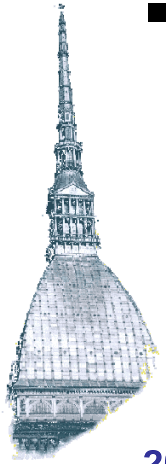


# Fits of $\phi_h$ and $P_T$ dependence

- PDF: MRST LO 2001
  *Phys. Lett.* ***B531*** *(2002) 216*

- FF: Kretzer
  *Phys. Rev.* ***D62*** *(2000) 054001*

- Parameters: Constant values



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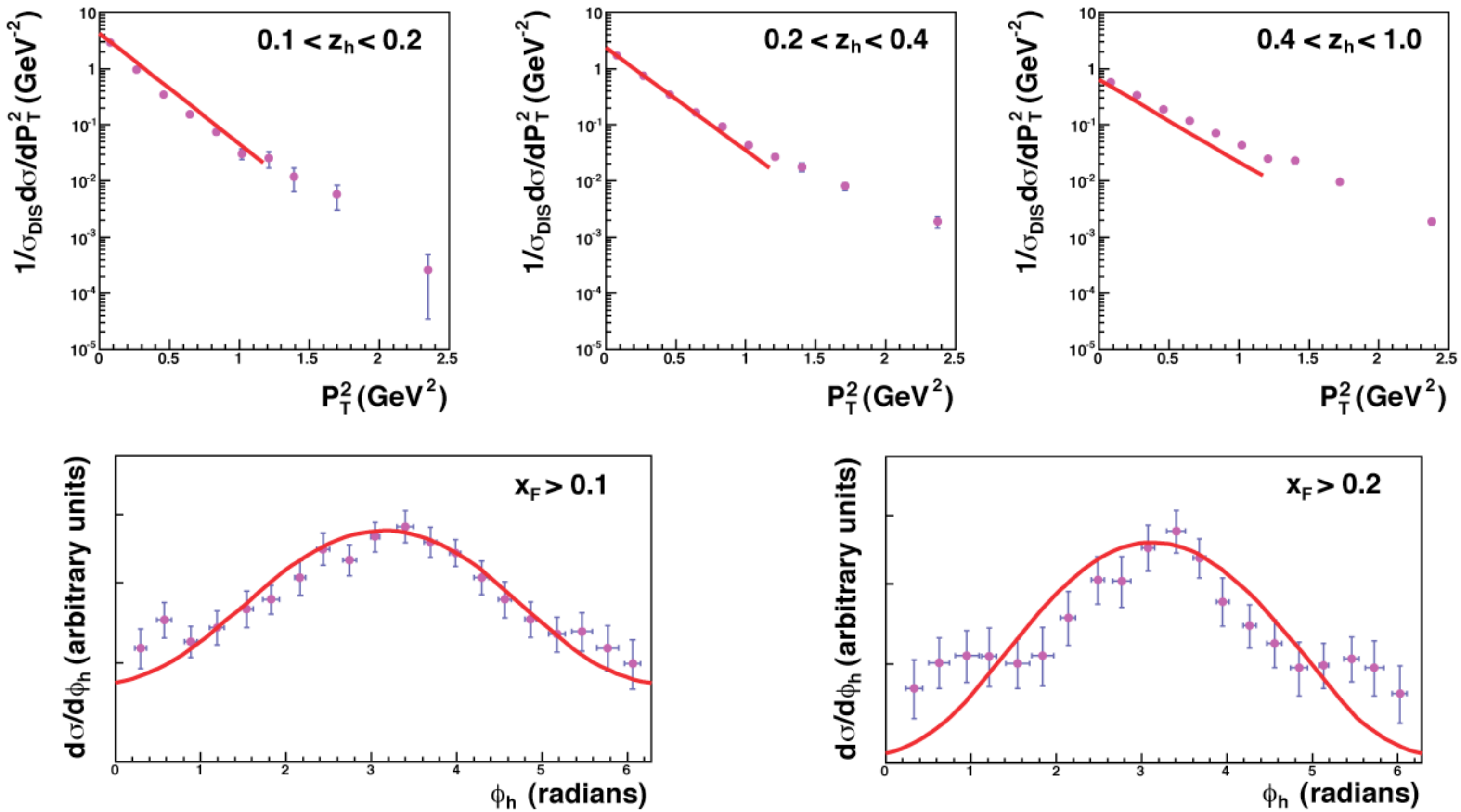
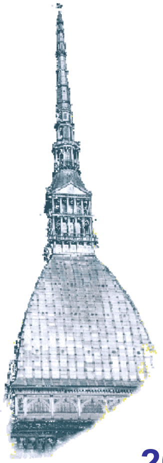
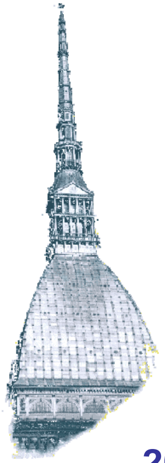


# Results and comments

**The best values of parameters**

$$\langle k_\perp^2 \rangle = 0.28 \ (\mathrm{GeV}^2) \quad \langle p_\perp^2 \rangle = 0.21 \ (\mathrm{GeV}^2)$$



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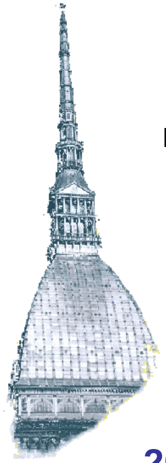
- Sets with different $x_b$, $Q^2$ and $z_h$ ranges are used and only constant parameters.
  - ✓ Introduce x and z dependence.

- Clearly valid for $P_T$ values up to 1 GeV.
  - ✓ NLO pQCD contributions must be considered.



# Planned and ongoing work

- Improvements in the parameters with x and z dependence.
- Include higher order corrections.
- Polarized SIDIS.
- …