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Interactions in nuclear matter with density-dependent couplings

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Contents:

- Introduction and motivation
- Model
- Relativistic Mean Field
- Results

Elements of an *ab initio* Relativistic Nuclear Field Theory

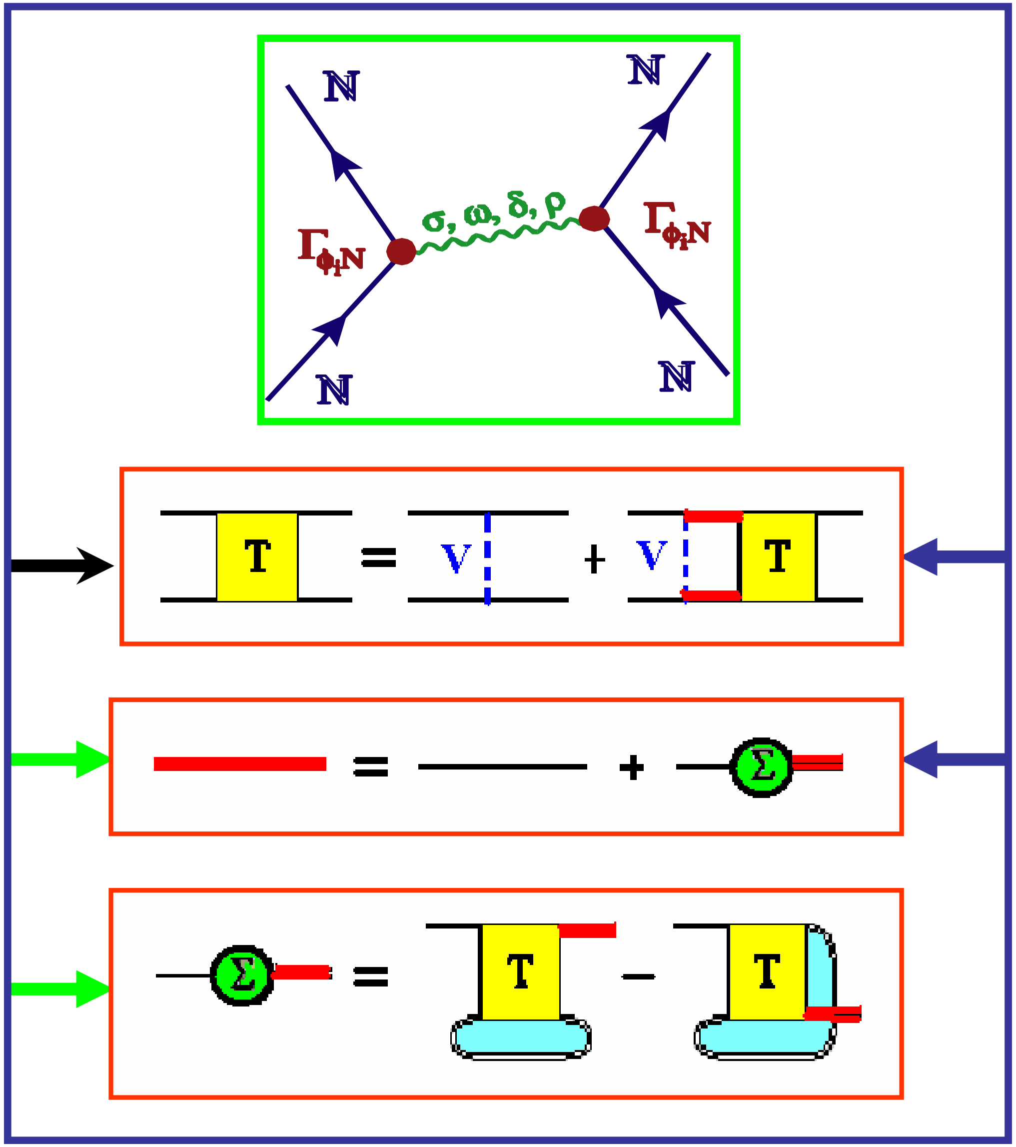
- meson exchange interactions

- free space and In-medium interactions from BS equation (Ladder Kernel)

- In-medium effects –

statistical: Pauli principle
dynamical: self-energies

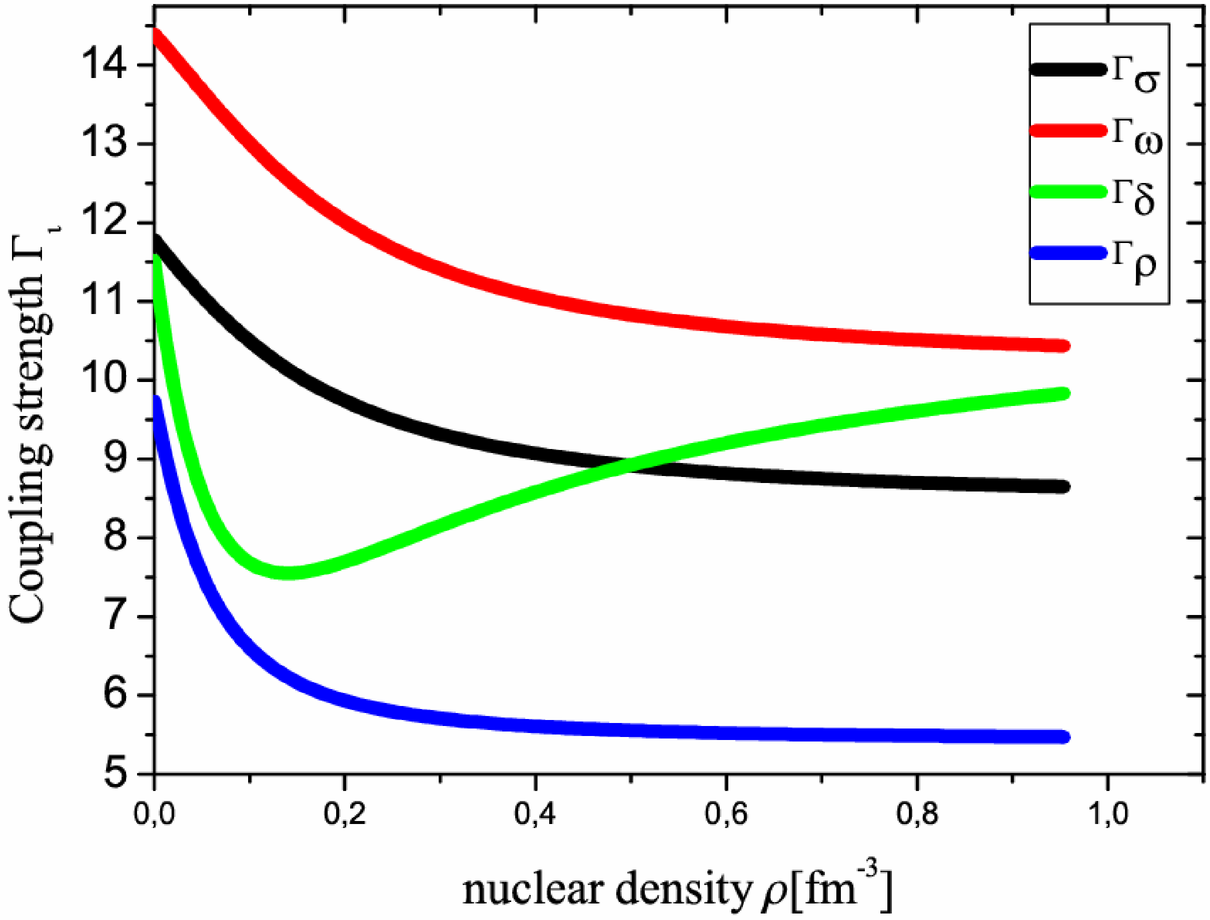
- Self-Consistent solution of Dyson and BS equations



PLB345 (1995), PRC52 (1995), PRC57 (1997), PRC64 (2001), Springer Lecture Notes (2004)

Motivation

Why is density-dependent coupling important ?



- …from where?
  - Microscopic Dirac-Brueckner calculations.
- Relativistic Mean Field Theory (Infinite nuclear matter)

The DDRH Lagrangian is

$$\mathcal{L} = \mathcal{L}_B + \mathcal{L}_M + \mathcal{L}_{int}$$

where:

- $\mathcal{L}_B$ **- the free baryonic**

$$\mathcal{L}_B = \bar{\Psi}(\imath\gamma_\mu\partial^\mu - M)\Psi$$

- $\mathcal{L}_M$ **- the free mesonic**

$$\begin{aligned}\mathcal{L}_M &= \frac{1}{2}\sum_{\imath=\sigma,\delta,\pi,\eta}\left(\partial_\mu\Phi_\imath\partial^\mu\Phi_\imath - m_\imath^2\Phi_\imath^2\right) - \\ &\frac{1}{2}\sum_{k=\omega,\rho,\gamma}\left(\frac{1}{2}F_{\mu\nu}^{(k)}F^{(k)\mu\nu} - m_k^2A_\mu^{(k)}A^{(k)\mu}\right)\end{aligned}$$

- $F_{\mu\nu}^{(k)}$**- the field strength tensor** (κ=ω, ρ,γ)

$$F_{\mu\nu}^{(k)} = \partial_\mu A_\nu^{(k)} - \partial_\nu A_\mu^{(k)}$$

- $\mathcal{L}_{int}$**- meson- baryon interactions**
- $\hat{Q}$**- electric charge operator**
- **Rearrangement self-energies**

$$\begin{aligned}\mathcal{L}_{int} &= \bar{\Psi}\hat{\Gamma}_\sigma(\hat{\rho})\Psi\Phi_\sigma - \bar{\Psi}\hat{\Gamma}_\omega(\hat{\rho})\gamma_\mu\Psi A^{(\omega)\mu} + \\ &\bar{\Psi}\hat{\Gamma}_\delta(\hat{\rho})\tau\Psi\mathbf{\Phi}_\delta - \bar{\Psi}\hat{\Gamma}_\rho(\hat{\rho})\gamma_\mu\tau\Psi\mathbf{A}^{(\rho)\mu} - \\ &e\bar{\Psi}\hat{Q}\gamma_\mu\Psi A^{(\gamma)\mu}\end{aligned}$$

… via the Euler-Lagrange equation… *Changes in equation of motion:*

- ✓ **mesons:**

$$(-\nabla^2 + m_\alpha^2)\Phi_\alpha = \hat{\Gamma}_\alpha(\hat{\rho})\rho_\alpha$$

- ✓ **baryons:**

…stays in structure unchanged

$$\left[\gamma_\mu\left(i\partial^\mu - \hat{\Sigma}^\mu\right) - \left(M - \hat{\Sigma}^s\right)\right]\Psi = 0$$

but additional terms occur:

rearrangement terms

$$\frac{\delta \mathcal{L}_{int}}{\delta \bar{\Psi}} = \frac{\partial \mathcal{L}_{int}}{\partial \bar{\Psi}} + \frac{\partial \mathcal{L}_{int}}{\partial \hat{\rho}}\frac{\delta \hat{\rho}}{\delta \bar{\Psi}}$$

❖ medium effects affect field dynamics in two ways:

- ✓ density dependent couplings,
- ✓ Baryonic field gets additional rearrangement self energies.

$$\hat{\Sigma}^{s(0)} = \hat{\Gamma}_\sigma(\hat{\rho})\Phi_\sigma + \hat{\Gamma}_\delta(\hat{\rho})\tau\Phi_\delta$$

$$\hat{\Sigma}^{\mu(0)} = \hat{\Gamma}_\omega(\hat{\rho})A^{(\omega)\mu} + \hat{\Gamma}_\rho(\hat{\rho})\tau\mathbf{A}^{(\rho)\mu} + e\hat{Q}A^{(\gamma)\mu}$$

$$\hat{\Sigma}^{\mu(r)} = \left(\frac{\partial\hat{\Gamma}_\omega}{\partial\hat{\rho}}A^{(\omega)\nu}\bar{\Psi}\gamma_\mu\Psi + \frac{\partial\hat{\Gamma}_\rho}{\partial\hat{\rho}}\mathbf{A}^{(\rho)\nu}\bar{\Psi}\gamma_\mu\tau\Psi - \frac{\partial\hat{\Gamma}_\sigma}{\partial\hat{\rho}}\Phi_\sigma\bar{\Psi}\Psi - \frac{\partial\hat{\Gamma}_\delta}{\partial\hat{\rho}}\Phi_\delta\bar{\Psi}\tau\Psi\right)\hat{u}^\mu.$$

- one gets

$$\hat{\Sigma}^{s} = \hat{\Sigma}^{s(0)} \qquad \hat{\Sigma}^{\mu} = \hat{\Sigma}^{\mu(0)} + \hat{\Sigma}^{\mu(r)}$$

- The effective baryon mass differs for p & n ←δ meson

$$M^* = M - \hat{\Sigma}^{s}$$

$$\hat{\Sigma}^{s} = \hat{\Sigma}^{s}_{\sigma} + \tau_3 \hat{\Sigma}^{s}_{\delta} \qquad \tau_3 = \left\{ \begin{array}{ll} +1, & p \\ -1, & n \end{array} \right\}$$

Special features of DDRH theory

- The meson-baryon vertices $\hat{\Gamma}_{\alpha}\ (\alpha = \sigma, \omega, \delta, \rho)$ are not constant numbers but functionals of the baryon field operators Ψ .
- Lorentz invariance and Relativistic covariance requires that the vertices $\hat{\Gamma}_{\alpha}(\hat{\rho})$ are functions of Lorentz-scalar bilinear forms $\hat{\rho}(\bar{\Psi}, \Psi)$ of the field operators.

- SDD !

$$\hat{\rho} = \bar{\Psi}\Psi$$

- VDD

$$\hat{\rho}^2 = \hat{j}_{\mu}\hat{j}^{\mu}$$

$$\hat{j}_{\mu} = \bar{\Psi}\gamma\Psi$$

(the baryon vector current)

Relativistic Mean Field Theory

- Investigating **ground state**:
  - ✓ Only the zero component of the vector fields contributes.
  - ✓ Time reversal symmetry is assumed.
- The meson fields are replaced by their expectation values.
- for infinite nuclear matter:
  - ✓ The electromagnetic field has to be neglected
- Due to translational and rotational invariance
  - ✓ The meson fields are treated as static classical fields.

$$\Phi_\sigma = \frac{\Gamma_\sigma(\rho)\rho^s}{m_\sigma^2}$$

  - ✓ Solutions of the stationary Dirac equation are usual plane wave DS.

$$\left[\gamma_\mu k_b^{*\mu} - m_b^*\right] u_b^*(k) = 0$$

here $k_b^{*\mu} = k_b^\mu - \Sigma_b^\mu \qquad m_b^* = M - \Sigma_b^s$

$$k_b^{*2} = m_b^{*2}$$

$$u_b^*(k) = \sqrt{\frac{E_b^* + m_b^*}{2m_b^*}} \begin{pmatrix} 1 \\ \frac{\sigma k_b^*}{E_b^* + m_b^*} \end{pmatrix} \chi_b$$

$$E_b^* = k_b^{*0} = \sqrt{k_{F_b}^{*2} + m_b^{*2}}$$

$$k \leq k_{F_b} \quad \rightarrow \quad E_{F_b} = \sqrt{k_{F_b}^2 + m_b^{*2}}$$

(T=0, Fermi energy)

- in Dirac-Brueckner calculations

$$\Sigma^{DB} \doteq \frac{\Gamma_\alpha^2(\rho)}{m_\alpha^2}\rho_\alpha$$

- in the RMF for the infinite nuclear matter self–energies :

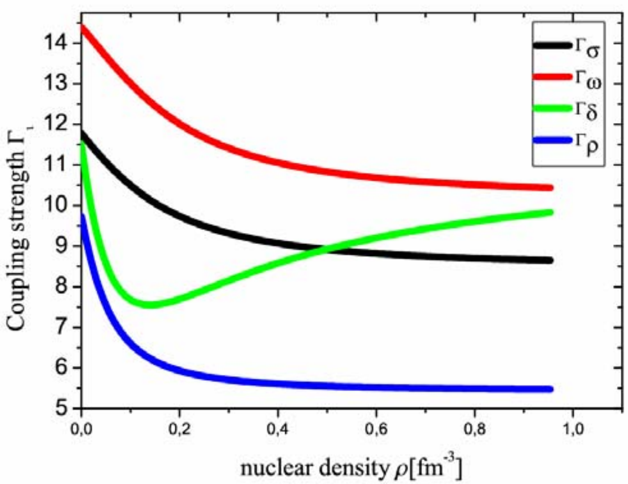
$$\Sigma^{MF} = \Gamma_\alpha \Phi_\sigma$$

$$\Phi_\alpha = \frac{\Gamma_\alpha(\rho)\rho_\alpha}{m_\alpha^2}$$

$$\boxed{\Sigma^{MF} \equiv \Sigma^{DB}}$$

... from Groningen NN potential

$$\Gamma_\alpha(\rho) = a_\alpha \frac{1 + b_\alpha(\frac{\rho}{\rho_0} + d_\alpha)^2}{1 + c_\alpha(\frac{\rho}{\rho_0} + e_\alpha)^2}$$



meson	σ	ω	δ	ρ
m_α [MeV]	550	783	983	770
a_α	13.1334	15.1640	19.1023	12.8373
b_α	0.4258	0.3474	1.3653	2.4822
c_α	0.6578	0.5152	2.3054	5.8681
d_α	0.7914	0.5989	0.0693	0.3671
e_α	0.7914	0.5989	0.5388	0.3598
$\Gamma(\rho=0)$	11.78187	14.39368	11.51854	9.735665
$\rho_0=0.16$ [fm^{-3}]	9.990312	12.35660	7.574376	6.108742

- ✓ Effective masses

$$\begin{cases} m_p^* = M - \Gamma_\sigma(\rho)\Phi_\sigma + \Gamma_\delta(\rho)\Phi_\delta \\ m_n^* = M - \Gamma_\sigma(\rho)\Phi_\sigma - \Gamma_\delta(\rho)\Phi_\delta \end{cases}$$

- ✓ The scalar and vector densities

$$\rho_b^s = \frac{2}{(2\pi)^3}\int_{|k|\le k_{F_b}} d^3k \frac{m_b^*}{E_b^*} = \frac{m_b^*}{2\pi^2}\left[k_{F_b}E_{F_b} + m_b^{*2}\ln\frac{k_{F_b}+E_{F_b}}{m_b^*}\right]$$

$$\rho_b = \frac{2}{(2\pi)^3}\int_{|k|\le k_{F_b}} d^3k = \frac{k_{F_b}^3}{3\pi^2}$$

- ✓ Energy density:

$$\begin{aligned}\mathcal{E} = \langle T^{00}\rangle &= \sum_{b=n,p}\frac{1}{4}\left[3E_{F_b}\rho_b + m_b^*\rho_b^s\right] \\ &+\frac{1}{2}\left[m_\sigma^2\Phi_\sigma^2 + m_\delta^2\Phi_\delta^2 + m_\omega^2 A_0^{(\omega)^2} + m_\rho^2 A_0^{(\rho)^2}\right] \\ &= \sum_{b=n,p}\frac{1}{4}\left[3E_{F_b}\rho_b + m_b^*\rho_b^s\right] + \sum_{b=n,p}\frac{1}{2}\left[\rho_b\Sigma_b^{0(0)} + \rho_b^s\Sigma_b^{s(0)}\right]\end{aligned}$$

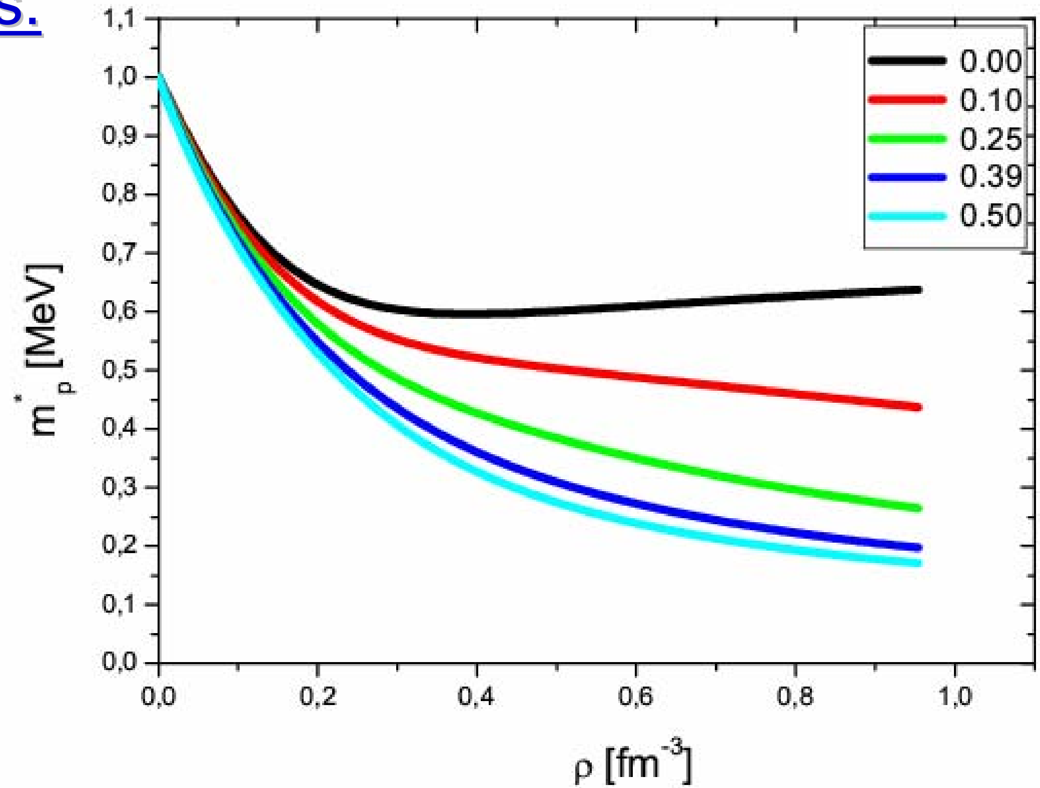
- ✓ Pressure:

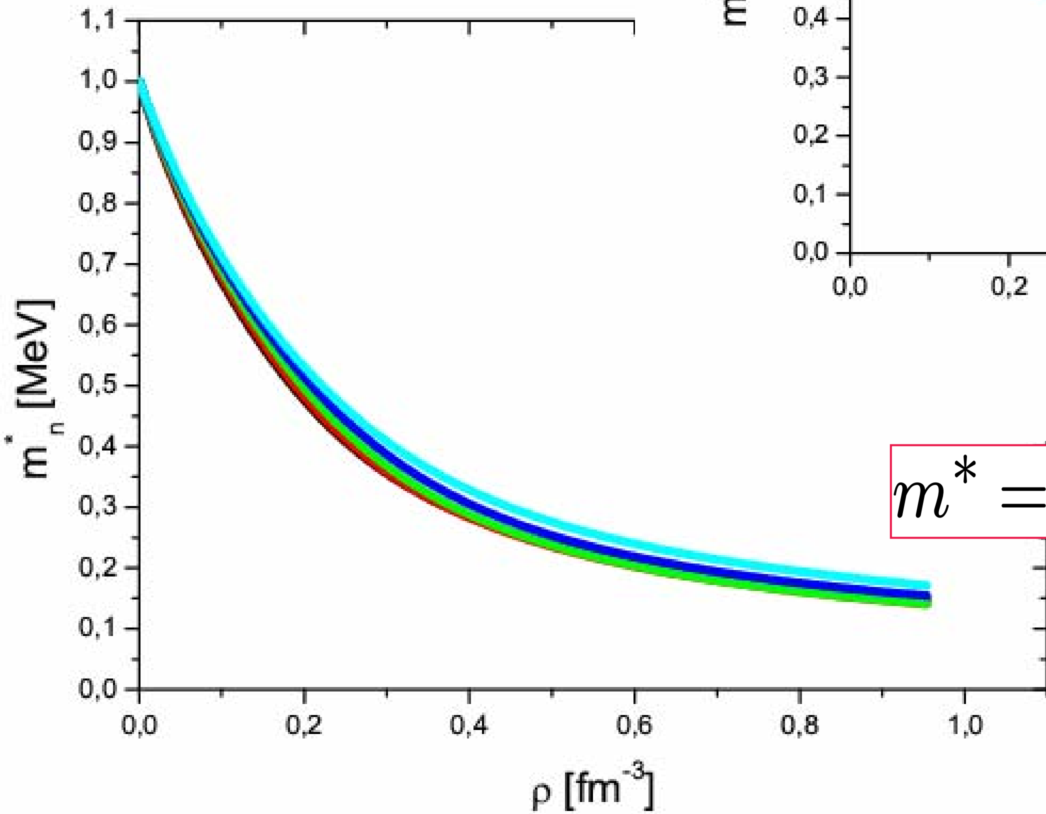
$$\begin{aligned}p = \frac{1}{3}\sum_{i=1}^{3}\langle T^{ii}\rangle &= \sum_{b=n,p}\frac{1}{4}\left[E_{F_b}\rho_b - m_b^*\rho_b^s\right] + \sum_{b=n,p}\rho_b\Sigma^{0(r)} \\ &-\frac{1}{2}\left[m_\sigma^2\Phi_\sigma^2 + m_\delta^2\Phi_\delta^2 + m_\omega^2 A_0^{(\omega)^2} + m_\rho^2 A_0^{(\rho)^2}\right] \\ &= \sum_{b=n,p}\frac{1}{4}\left[E_{F_b}\rho_b - m_b^*\rho_b^s\right] + \rho\Sigma^{0(r)} + \sum_{b=n,p}\frac{1}{2}\left[\rho_b\Sigma_b^{0(0)} - \rho_b^s\Sigma_b^{s(0)}\right]\end{aligned}$$

- ❖ Thermodynamically consistent

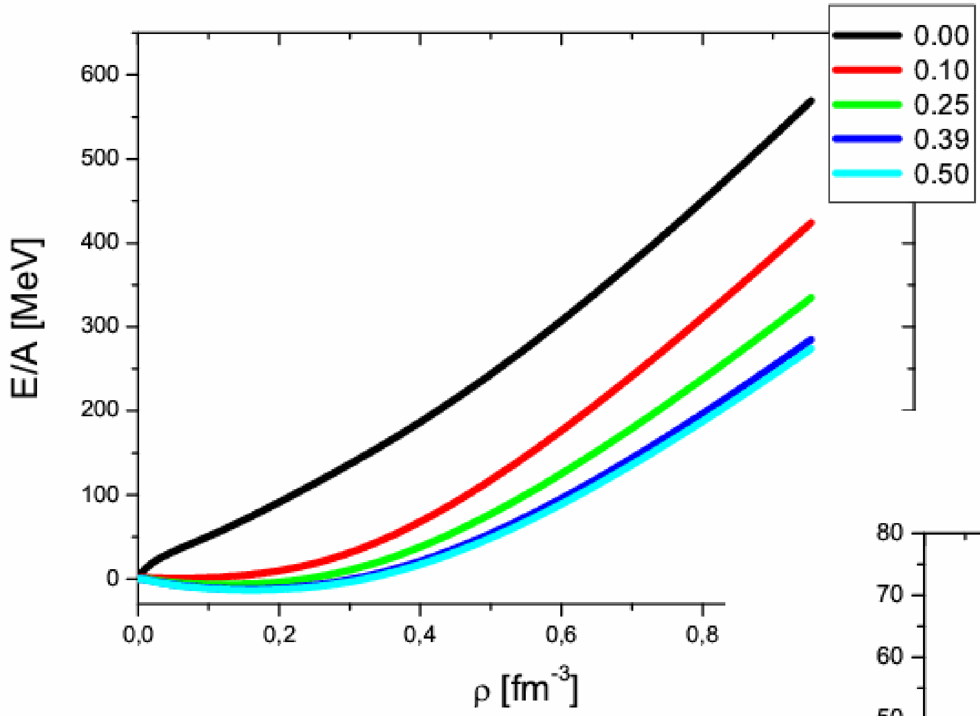
$$p = \rho_b^2\frac{\partial}{\partial\rho_b}\left(\frac{\mathcal{E}}{\rho_b}\right)$$

Effective nucleon masses:

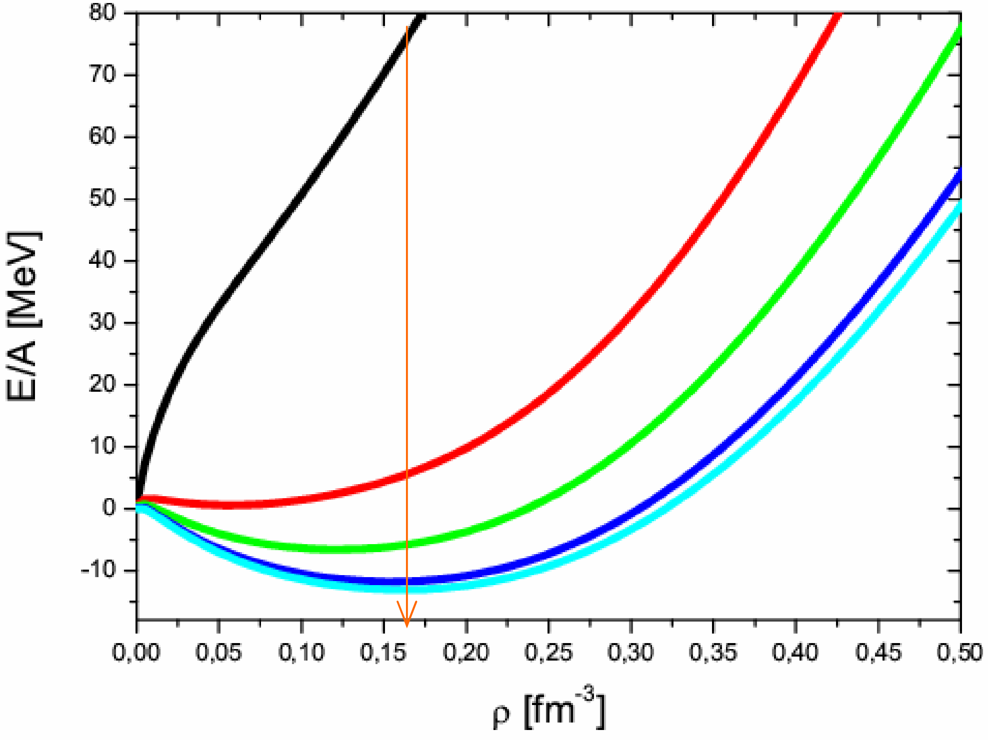




$$m^* = M - \Gamma_\sigma(\rho)\Phi_\sigma \pm \Gamma_\delta(\rho)\Phi_\delta$$



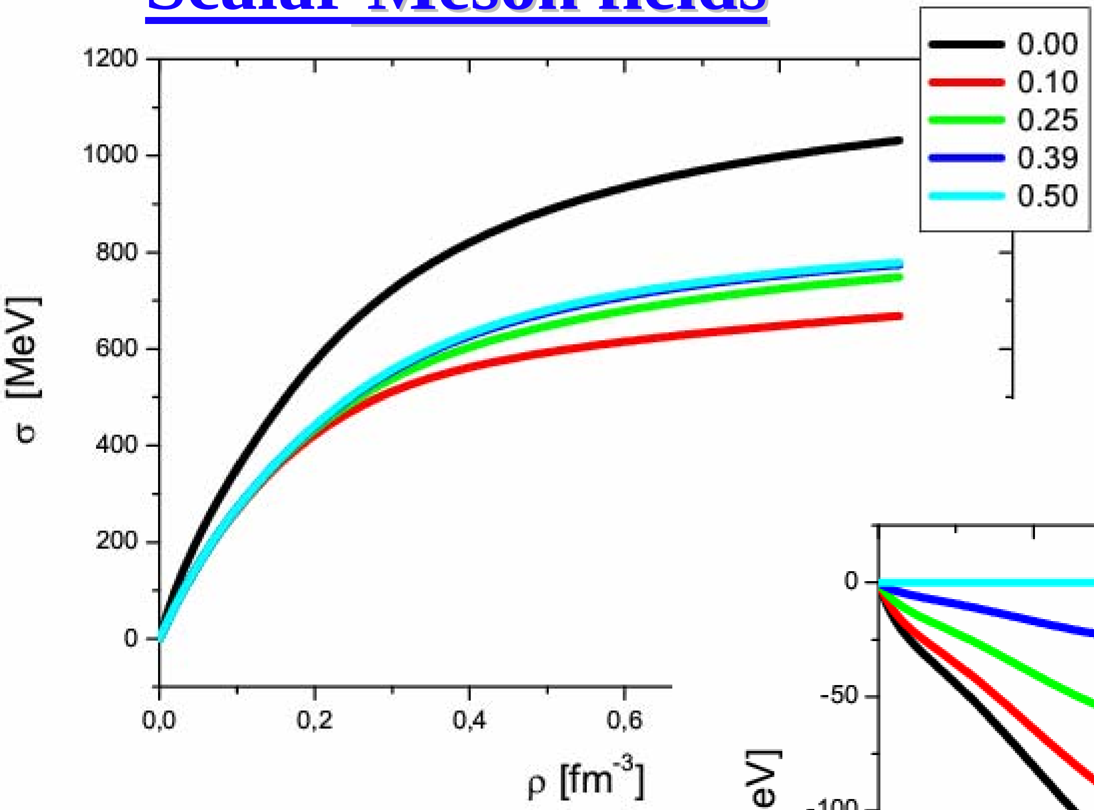
Equation of State



… symmetric nuclear matter

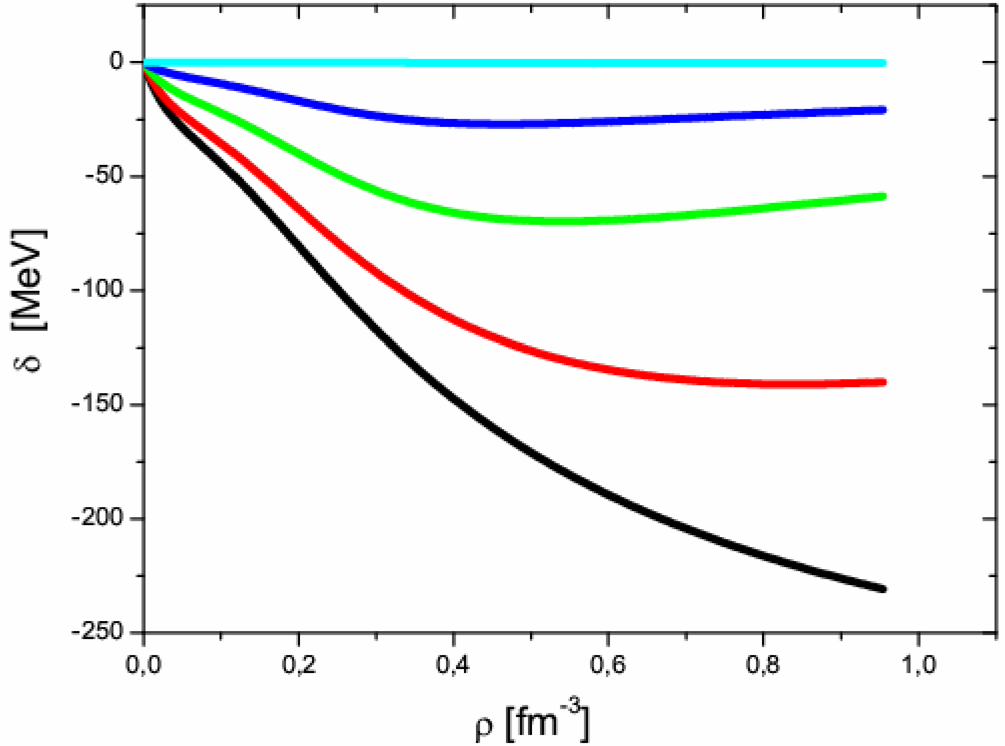
ρ_{sat} [fm^{-3}]	0.161
E/A [MeV]	-13.13
p [MeV/ fm^3]	0.0001
K [MeV]	211.34
α_4 [MeV]	28.90

Scalar Meson fields



$$\Phi_\sigma = \frac{\Gamma_\sigma(\rho)\rho^s}{m_\sigma^2}$$

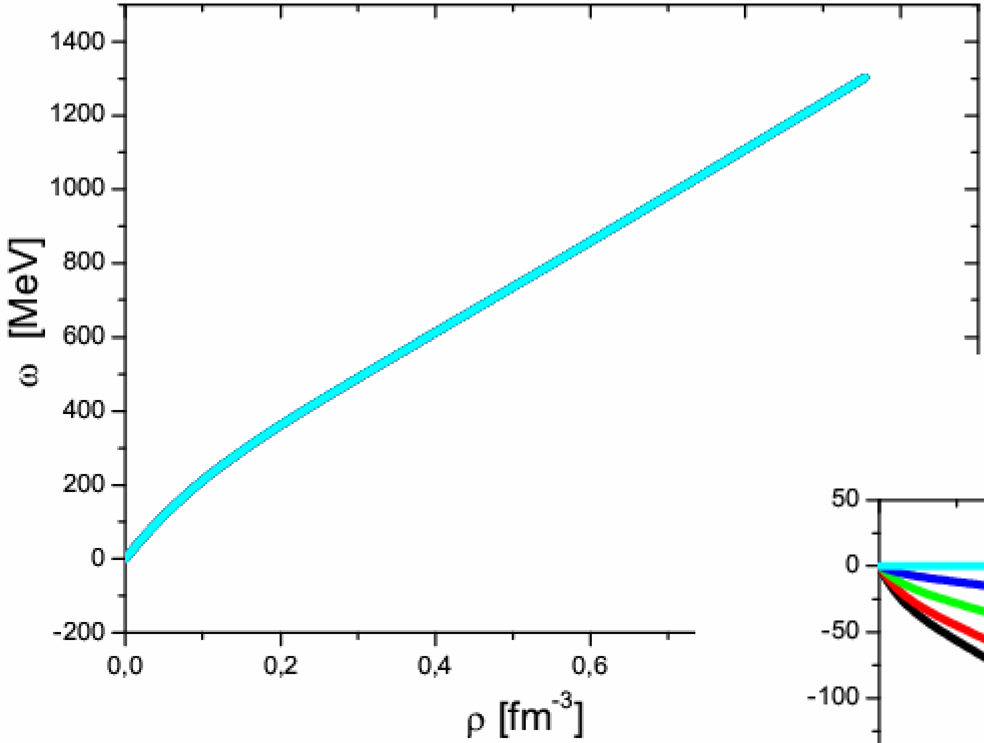
$$\rho^s = \langle\bar{\Psi}\Psi\rangle = \rho_n^s + \rho_p^s$$



$$\Phi_\delta = \frac{\Gamma_\delta(\rho)\rho_3^s}{m_\delta^2}$$

$$\rho_3^s = \langle\bar{\Psi}\tau_3\Psi\rangle = \rho_n^s - \rho_p^s$$

Vector Meson fields

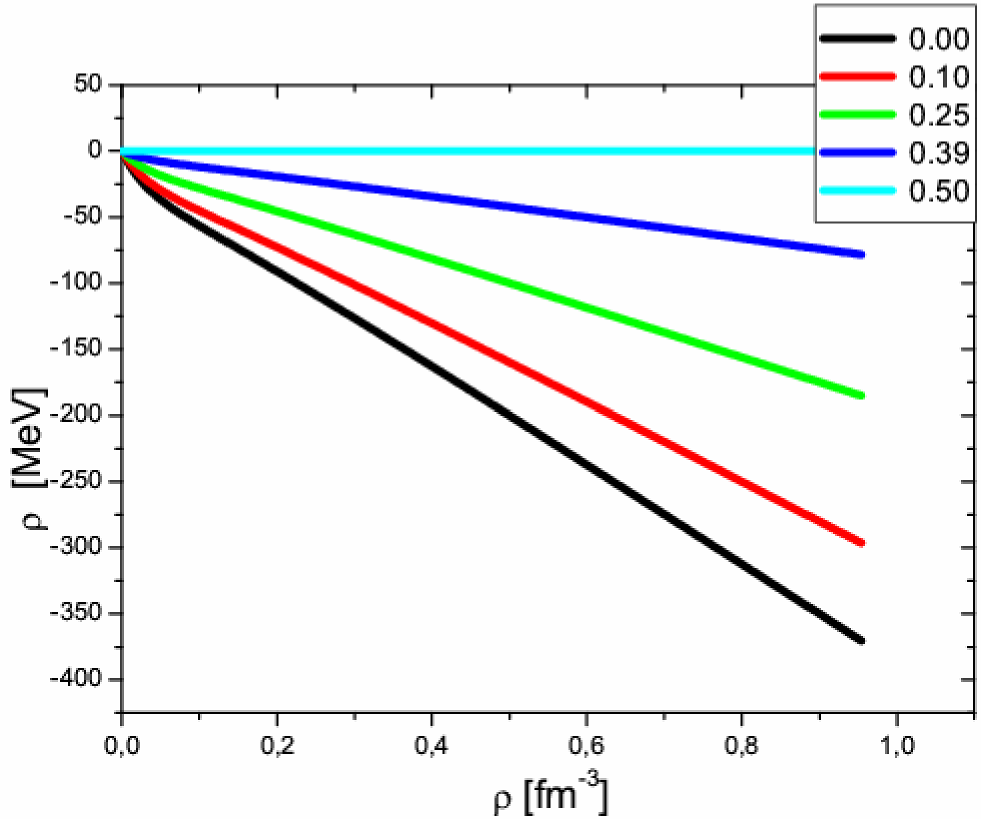


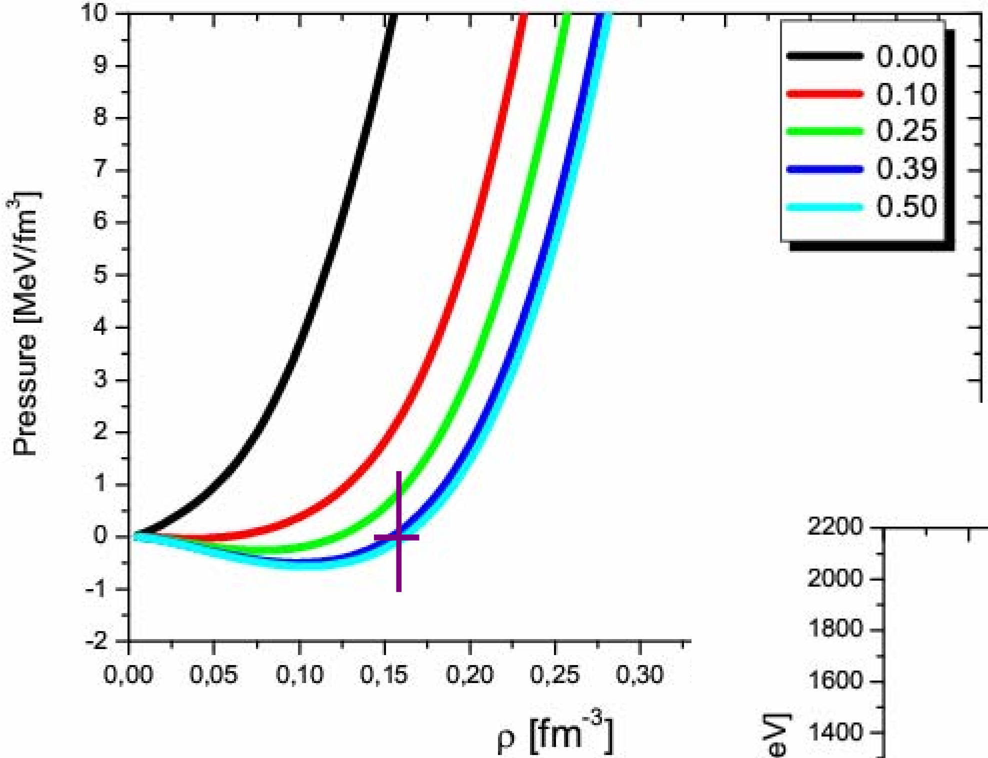
$$\Phi_\omega = \frac{\Gamma_\omega(\rho)\rho}{m_\omega^2}$$

$$\rho = \langle \bar{\Psi}\gamma_0\Psi \rangle = \rho_n + \rho_p$$

$$\Phi_\rho = \frac{\Gamma_\rho(\rho)\rho_3}{m_\rho^2}$$

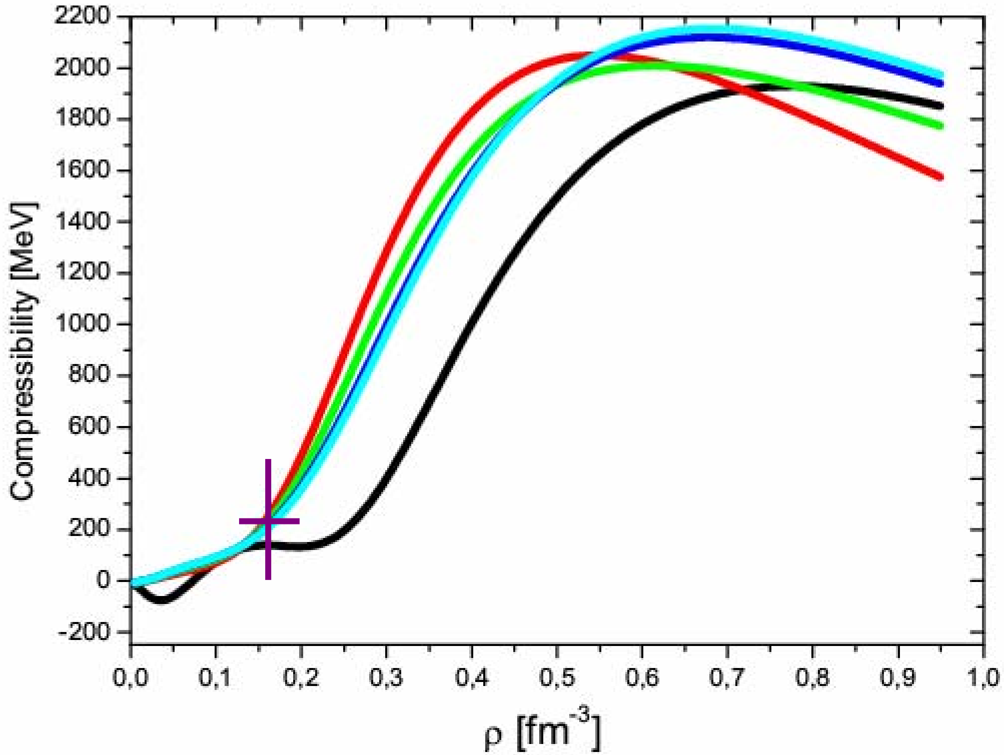
$$\rho_3 = \langle \bar{\Psi}\gamma_0\tau_3\Psi \rangle = \rho_n - \rho_p$$





Pressure

$$p = \rho^2 \frac{\partial}{\partial \rho}\left(\frac{\mathcal{E}}{\rho}\right)$$



Compressibility

$$K_\infty = 9\rho^2 \frac{\partial^2}{\partial \rho^2}\left(\frac{\mathcal{E}}{\rho}\right)$$

$$K_\infty = K_0 + K_1$$

Summary and Outlook

- *ab initio* description of RMF interactions
- EoS for symmetric and asymmetric nuclear matter
- Isovector effective masses
- Scalar and vector Mean fields

- Excitations in infinite nuclear matter (Loop diagrams)
  - Dynamical correlations (Sunset diagrams)
  - Neutron stars